A Reconciliation of Taguchi's Robustness and Suh's Minimum Information Axiom

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Abstract

Since Taguchi introduced robust design to US, design quality has drawn continuous attention from quality experts, engineers and statisticians. Taguchi used signal-to-noise ratio to measure the robustness of a design. Based on engineering principles, Suh (1990) proposed the independence axiom and the minimum information axiom as a "frame of reference" for comparing the merits of competing designs. According to Suh, the axiomatic approach appeals directly to an engineer's heuristics; nevertheless, it seems to lack a well-defined data procedure. On the other hand, many engineers still need a justification to use Taguchi's prescriptions, because an explanation consistent with engineering rationals has not yet appeared.

We will show that Suh and Taguchi share the same engineering objective, but differ in their data strategies. Both methods have their respective relevancies in different stages of a design task. Suh's information measures the absolute design merit, whereas Taguchi's signal-to-noise ratio measures the relative consistency. Hence, in the design iteration stage, signal-to-noise ratio is more effective. After the design validation stage, where the distribution of design performance is formulated, Suh's information can be computed. Information is critical in predicting the reliability of a design.

We introduce a notion of unscaled dispersion. It provides a common ground so that two method can be compared. The reconciliation in objective clarifies the notion of quality and justifies signal-to-noise ratio.

Key Words: Functional requirement; design parameters; data partition; bias; intrinsic dispersion; dimensional analysis; signal-to-noise ratio; and information.

1 Introduction

Since Taguchi introduced robust design to US, design quality has drawn continuous attention from statisticians, quality experts and engineers. Taguchi emphasizes that the final quality of a product/process is more a virtue of good design than on-line inspection or quality control. Therefore, good design ensures high quality; the design engineer becomes the first line of defense against poor quality. Taguchi proposed signal-to-noise ratio, hereafter called SN ratio, to measure design quality. A design with a high SN ratio is robust against all the manufacturing/usage perturbations; hence, it preserves the functionality of the design with good fidelity. Two of Taguchi's contributions are quantifying quality via SN ratio and devising operationally feasible steps to maximize SN ratio.

Suh, Bell and Gossard (1978), three engineers from MIT, proposed principles for evaluating design merits. In contrast to Taguchi's statistical approach, Suh, Bell and Gossard took an axiomatic approach. Later, those principles were condensed by Suh (1990) into two design axioms: independence axiom and minimum information axiom, which, he suggested, serve as an "absolute frame of reference". Suh believes that the axiomatic approach complements the traditional algorithmic approach. Suh stated that these two axioms "will always lead to good results and they will so narrow the range of possibilities". In Suh's context, the less the dependence among its functions, the better the design; also, the less the information required, the better the design.

According to Suh, the axiomatic approach appeals directly to an engineer's heuristics; nevertheless, it seems to lack a well-defined data procedure. On the other hand, many engineers still need a justification to use Taguchi's prescriptions, because an explanation consistent with engineering rationals has not yet appeared. We will show that Suh and Taguchi share the same objective, but differ in their data strategies.

Both methods have their respective relevancies in different stages of a design task. The reconciliation in objective clarifies the notion of quality and justifies the SN ratio. It also heightens the differences of the two methods. In fact, Suh (1990) recognized the alignment of the two methods. In his book, he demonstrated Taguchi method via examples and claimed that better robustness (higher value of SN ratio) would lead to less information. However, the connection was not theoretically demonstrated, because Suh's information is defined in terms of an mostly unobservable distribution and Taguchi's SN ratio is calculated directly from the observed data. Their differences must be reconciled on two fronts: the qualitative definition and the quantitative measures of quality. In this paper, we will show that Suh's information and Taguchi's robustness are measures related to *reproducibility*, the probability to reproduce the design objectives.

In Section 2, a brief discussion of Suh's design axioms and Taguchi's SN ratio will be presented. The reconciliation and comparison are derived in Section 3. We show that SN ratio is equivalent to a measure of unscaled dispersion, which is invented to circumvent the incomparable scales of data. Unscaled dispersion provides a common ground, such that these two methods can converge. We also discuss the impact of bias on information and the advantage of removing bias from dispersion. In Section 4, we state how these two methods compliment each other: SN ratio, combined with design of experiment, is most appropriate for design iterations; Suh's information, however, is the definite benchmark for design acceptance.

2 Suh's design axioms and Taguchi's measure of robustness

Based on engineering principles, Suh (1990) proposed independence axiom and minimum information axiom as a "frame of reference" for comparing the merits of competing designs. It is, however, only meaningful to compare designs serving the same functionality. Function requirements (FRs) are defined as the minimum set of requirements which completely characterizes the functional needs of a product/process. By this definition, one FR can not substitute for any other FRs. Let the space in which FRs reside be called the *functional domain*. The design parameters (DPs) are the set of physical entities created by the designer to fulfill the FRs. Let the space in which DPs reside be called the *design domain*, and it consists of all the physical options available to the design engineers or users. For example, the FRs of a faucet are water volume and temperature selections and the user's DPs are the two orthogonal movements of the faucet knob. Also, separate buttons on a TV remote control provide accesses to the FRs of on/off, volume, picture brightness, channel, etc. The physical bottoms are the DPs. Naturally, there exists a mapping between DPs and FRs. The main tasks of a designer are to select the proper DPs and to formulate an efficient mapping between DPs and FRs.

Independence axiom requires that the perturbation of a particular DP must affect only its referent FR, not any other FRs. Hence, for a design to satisfy independence axiom, modification made to any DP should only impact its corresponding FR. For example, if a faucet satisfies the independence axiom, then an adjustment to increase water volume should have no impact on water temperature. Independence axiom is also observed by most TV remote controls.

Minimum information axiom states that among the designs satisfying independence axiom, the optimal design is the one that requires the least information. More specifically, for any FR, say y, let $\mathcal{R}$ be the range and $f(y)$ be the density function of y. Suh called $f(y)$ the *FR distribution*, which represents the variability of the FR in real production/usage environments. Let c be the target value for y and the *design* *tolerance* of y is the interval

$$\mathcal{L} = (c - \Delta_1, c + \Delta_2),$$

where $\mathcal{L}$ is also called the *design specification*. The mean of y is defined as

$$m = \int_{\mathcal{R}} y f(y) dy.$$

The difference between m and c is called the bias. When m is equal to c, the FR distribution is called *unbiased*. In the past, most design objectives concentrated on making the design unbiased; relatively few engineers investigated the variability of y. Both Suh and Taguchi, nevertheless, believe that true merit should be judged by the probability that a FR meet its design tolerance. Suh called it the probability of success and Taguchi referred to it as the *reproducibility* (Japanese:) of a design in real production. Mathematically, it is defined as

$$p = \int_{\mathcal{R} \bigcap \mathcal{L}} f(y) dy.$$

Suh defined information as

$$I = -\log p.$$

The best scenario is that $\mathcal{R}$ is a subset of $\mathcal{L}$. Then p equals 1 and I equals zero. No extra information is required to make all the y meet tolerance. The worst scenario is that $\mathcal{R}$ does not intersect with $\mathcal{L}$. Under this circumstance, none of the output y meet the design specification and information becomes infinity. Suh's terminology is derived from the perspective of a process design engineer. Using Suh's terminology, when only $l00p$ percentage of the outputs meet the specifications, then an extra of $-\log p$ amount of information is needed to make 100 percent of the output fall inside $\mathcal{L}$. Naturally, the larger the p, the smaller the information, and the better the design.

From the definition of p, there are two possible ways to increase p :

1. to shrink range $\mathcal{R}$, which implies less variability for y; or

2. to expand $\mathcal{L}$, which is equivalent to allowing the FR to have more variability.

It is, hence, obvious that Suh's definition of design merit is closely connected with the dispersion of y.

While Suh's information is defined via $\mathcal{R}$ and $\mathcal{L}$, Taguchi's measure of robustness reflects only the variability of y in $\mathcal{R}$, without referencing $\mathcal{L}$. Through a conscientious introduction of noise variables into the design laboratory, let $\{y_1, y_2, \cdots, y_n\}$ be a representative sample of $f(x)$. Let $\bar{y} = \sum_{i=1}^{n} y_i/n$ and $s^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2/(n-1)$ be the sample mean and variance. Then, SN ratio is defined as

$$\eta = 10 \log_{10}(\bar{y}^2/s^2).$$

One argument in favor of a larger value of η is it represents a smaller value for s^2. This leads some statisticians to minimize s^2 directly, instead of maximizing η. However, s^2 and η represent very different objectives, as will be shown in Section 3.

In general, different FRs take different physical measurements, such as hardness, smoothness, length, weight, etc.. These quantities are said to have different dimensions as Dimensional Analysis (see Langhaar (1980)) refers to them. For a design task involving multiple FRs, the variances of multiple FRs will have different dimensions and, hence, cannot be combined into a single measure to reflect the combined virtues for the entire task. Quantities with different dimensions can not be compared either; for example, a comparison can not be made between length and smoothness. This is one major reason why Suh derived his measure of quality from probability, because, regardless of the original dimension of y, the probability p is always dimensionless; so is information I. When multiple FRs are independent, probabilities become multiplicative and information can be added up to show the total amount of information required for the entire task. Only dimensionless quantities can be combined and compared without restriction; in this regard, any measure of design virtue better be dimensionless. With regard to SN ratio, since $\bar{y}$ and s have the same dimension, their ratio is always dimensionless, so is η.

After knowing that η has no dimension and s has the same dimension as y, it become immediately clear that the relationship between them can not be one-to-one or monotonic. Increasing value of η does not necessarily reduce s^2. For sample $\{y_1, y_2, \cdots, y_n\}$, the empirical value of reproducibility is

$$\begin{aligned} \hat{p} &= Pr\{c - \Delta_1 \le y_i \le c + \Delta_2 \ \text{ for } \ 1 \le i \le n\} \\ &= Pr\{\frac{1}{n} - \frac{\Delta_1}{nc} \le \frac{y_i}{nc} \le \frac{1}{n} + \frac{\Delta_2}{nc} \ \text{ for } \ 1 \le i \le n\} \\ &= Pr\{\frac{1}{n} - \frac{\Delta_1}{nc} \le \frac{y_i}{T}\frac{T}{nc} \le \frac{1}{n} + \frac{\Delta_2}{nc} \ \text{ for } \ 1 \le i \le n\}, \end{aligned}$$

where $T = y_1 + y_2 + \cdots + y_n$ is the total of the sample.

When the sample is unbiased, i.e. $T = nc$, the above probability becomes

$$\hat{p} = Pr\{-\frac{\Delta_1}{nc} \le (\frac{y_i}{T} - \frac{1}{n}) \le \frac{\Delta_2}{nc} \ \text{ for } \ 1 \le i \le n\}.$$

In many cases, Δ_1 equals to Δ_2 and the above equations becomes

$$\hat{p} = Pr\{\left|\frac{y_i}{T} - \frac{1}{n}\right| \le \frac{\Delta_1}{nc} \ \text{ for } \ 1 \le i \le n\}.$$

Shortening the "distance" between two n-dimensional points: $(\frac{y_1}{T}, \cdots, \frac{y_n}{T})$ and $(\frac{1}{n}, \cdots, \frac{1}{n})$ increases $\hat{p}$. A convenient statistic for distance is :

$$\sum_{i=1}^{n}(\frac{y_i}{T} - \frac{1}{n})^2 = \frac{1}{T^2}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{n-1}{n^2}\frac{s^2}{\bar{y}^2},$$

which is a monotonic function of SN ratio.

We have shown the following:

1. Both information and SN ratio are dimensionless, which makes combining merits of multiple FRs possible.

2. Both information and reproducibility are monotone functions of SN ratio, provided the distribution of FR is unbiased.

In brief, when $m = c$, Taguchi and Suh share the same objective and use mathematically equivalent statistics to reflect the design merit. In the next section, we will consider the biased case, where $m \neq c$ and show how a small bias can diminish reproducibility and render the direct comparison of reproducibilities inconclusive.

3 Intrinsic dispersion and bias

Since data $\{y_1, y_2, \cdots, y_n\}$ are not always unbiased, there is an advantage to separate the mean from the data. One way is to partition $\{y_1, y_2, \cdots, y_n\}$ into $\{T = \sum y_i\}$ and $(y_1 - \bar{y}, \cdots, y_n - \bar{y})$, which leads to sample mean, $\bar{y}$, and sample variance s^2. Assuming all y's are positive, another way to partition the sample is as follows:

$$\psi : \{y_1, y_2, \cdots, y_n\} \longrightarrow \{T = \sum y_i\} \quad \text{and} \quad \{(\frac{y_1}{T}, \cdots, \frac{y_n}{T})\}, \tag{3.1}$$

and the amount of departure of $\frac{T}{nc} = \frac{\bar{y}}{c}$ from 1 measures the degree of biasedness. The array $(\frac{y_1}{T}, \cdots, \frac{y_n}{T})$ is called the *intrinsic dispersion vector*. Statistic $\sum_{i=1}^{n}(\frac{y_i}{T} - \frac{1}{n})^2 = (\frac{n-1}{n^2})\frac{s^2}{\bar{y}^2}$ is a measure of the departure of intrinsic dispersion vector from $(\frac{1}{n}, \cdots, \frac{1}{n})$. There are two comparisons to be discussed:

a) Suh's information vs. SN ratio, where bias plays a significant role; and

b) the intrinsic dispersion vs. common dispersion s^2.

3.1 Suh's information vs. Taguchi's SN ratio

Figure 3.1 represents figurative shots toward a bull's-eye target made by Mr. Smith and Mr. Dole, respectively. Out of 10 shots fired, Mr. Smith had eight shots inside the bull's-eye, while Mr. Dole had none. Let the bull's-eye circle be the design tolerance; then, the probabilities are 0.8 and 0, and Suh's information is 0.322 and infinity, respectively, for Mr. Smith and Mr. Dole. Using Suh's information as the criterion, Mr. Smith is judged the better of the two. However, Taguchi and Clausing (1990) considered Mr. Dole's shooting as "consistent and predictable. A small adjustment in his sight will give him many perfect bull's-eyes in the next round."

This discrepancy is caused by the bias and can be remedied via data partition. Suh's information depends on the data in its original measurements and, hence, confounds bias and dispersion in one statistic; Taguchi's SN ratio depends only on the intrinsic dispersion vector, free from the influence of bias. When bias is large, reproducibility inevitably approaches zero and Suh's information approaches infinity. Moreover, two FR distributions have the same bias, the one with the larger dispersion can have better reproducibility! Consider two uniform distributions over $(2, 4)$ and $(1, 5)$. If the design range is $(2-1, 2+0.5)$, then the less disperse FR has $p = 0.25$, while the more disperse FR has $p = 0.375$.

A similar scenario was described in Suh (1990, P.307), which compared six automobiles based on fuel efficiency, acceleration, noise level, trunk space and price. The ranges of fuel efficiency are $(11.0, 21.8)$ and $(8.7, 16.7)$ for car C and car E, respectively. The design requirement for gas mileage is 17 Km/L or more. Suh's information are 0.811 and infinity, respectively, for car C and car E. Nevertheless, car E had the shortest fuel efficiency range among all six cars compared; its gas mileage is more predictable than any of the other cars. No matter how favorable car E was according to the other criteria, it was automatically disqualified because its information on fuel efficiency was infinity. Based on the data presented there, it may be conjectured that car E is a heavier luxury car and, hence, delivered below average fuel efficiency. If its weight could be reduced, car E might deliver the most consistent gas mileage over 17 Km/L. The infinite information of fuel efficiency disproportionately diminishes the other merits of car E. This example clearly demonstrates that $\bar{y}$ profoundly overshadows dispersion when reproducibility is used as the criterion. This observation raises some reservations about the adequacy for using information to reflect the relative merit of a design, when the FR distribution is biased.

This property of Suh's information is by no means rare. In the design stage, a considerable number of designs will be experimented. It is highly likely that many of the FR distributions will fall far from the design tolerance and, hence, are biased. For example, the 27 samples generated by a passive filter circuit design are all outside the design tolerance, see Suh (1990, P.180 Table f). Due to the large sample size, it can be concluded that the entire FR distribution is out of the tolerance. This results in infinite information; consequently, these 27 samples offer little guidance about the relative merit as well as the direction for design improvement. In contrast, SN ratio always provides a continuous indication of the FR consistency.

3.2 SN ratio vs. variance

The debate between η and s^2 had been long and intense among statisticians. Leon, Shoemaker and Karcker (1987) proved that SN ratio is the ideal statistic when both quadratic loss and a multiplicative model are assumed for y. Their justification is strictly mathematical. For practical reasons, engineers demand a statistic which can consistently demonstrate the relative merits of competing designs, as Suh (1990) stated that "the exact form of the definition of information is not important, as long as it is an accurate predictor of relative complexity". In order to render its usefulness in design optimization, the summary statistics must be able to reflect the individual merit of a design but, more critically, can also make the collective data of competing designs comparable.

To illustrate the issue of the comparableness, we consider the data in Table 3.1.

In this example, we are dealing with a digital circuit which regulates the vertical display of a SVGA PC monitor. To gain efficiency in experiment, nine configurations of the same circuits are planned simultaneously at the onset of the experiment. For each configuration, the FR distribution is sampled at seven different combinations of noise. The measurements are the amount of vertical distortions. Sample mean, variance and SN ratio are also listed in Table 3.1. The design tolerance is 5 or less, i.e. $(0, 5)$. From Table 3.1, it is obvious that the experimental designs make no effort to meet the design tolerance. In stead, the objective of the experiments is to investigate the relationship between the DPs and the FR distributions.

The sample means and variances range from 2.04 to 8.30 and from 0.58 to 7.95, respectively. If minimum s^2 were used as the criterion, the third design with $s^2 = 0.58$ would be the best. In Figure 3.2, the standard deviations are plotted against the means. Except for the sixth and ninth designs, the mean-standard deviation points lie closely to a straight line. In fact, the coefficient of determination of a linear regression of s_i on $\bar{y}_i$ is 0.716; this rises to 0.9742 when the sixth and ninth designs are removed from the linear regression. Excluding the sixth and the ninth designs, the strong correlation between mean and standard deviation is indisputable. The smallest variance, 0.58, is associated with the smallest mean, 2.04. The next two smallest variances also associate with the next two smallest means. Under this correlation, it is hard to determine whether an observed small variance is caused by a small mean or by a consistent FR distribution. If mean does influence variance, then a direct comparison of variances becomes an indirect comparison of means. The confounding of mean and variance renders variance comparison unsatisfactory. How can a "uncontaminated" comparison of FR consistency be made without the hidden effect caused by the means? We need to identify the most likely mechanism which creates the coupling between means and variances. The answer resides in the scale of the FR distribution, because only scale can simultaneously affect mean and standard deviation in such a predictably linear fashion. Needless to say, if the scale of every FR distribution could be known or estimated, then each data set would be divided by its respective scale. After the scales had been equalized, comparison of dispersions would become possible. However, the scales are not available.

How to equalize the scales of the two data sets, say $\{y_1, y_2, \cdots, y_n\}$ and $\{z_1, z_2, \cdots, z_n\}$? When each y_i is inflated by a factor, say a, the total $T = \sum y_i$ will also be inflated by the same factor a. Hence, y and T must have the same scale. Consequently, their ratio becomes unscaled and dimensionless. Figure 3.3 represents how the partition ψ of (3.1) separates the dimensional and scaled part from the dimensionless and unscaled part. Note, also, that the total of any intrinsic dispersion vector, say $\sum \frac{y_i}{T_y}$ or $\sum \frac{z_i}{T_z}$, is always equal to one, regardless of the size, scale or dimension of the original measurements. This is further evidence that the scale comparableness among intrinsic dispersion vectors has been achieved.

To express the merit of a design, it has long been established that the consistency of the FR distribution is a critical criterion. The best scenario of consistency is that FR distribution is degenerated with no dispersion; the corresponding sample scenario is that all the measurements are equal, i.e. $y_1 = \cdots = y_n$. For the unscaled vector $(\frac{y_1}{T}, \cdots, \frac{y_n}{T})$, the corresponding ideal scenario is $(\frac{1}{n}, \cdots, \frac{1}{n})$. Hence, a summary for the intrinsic dispersion measures the "distance" between the intrinsic dispersion vector and $(\frac{1}{n}, \cdots, \frac{1}{n})$. In general, the statistics take the form of

$$\sum_{i=1}^{n} g(\left|\frac{y_i}{T} - \frac{1}{n}\right|) \tag{3.2}$$

where g is a convex function. SN ratio is a monotonically decreasing function of (3.2) with $g(x) = x^2$.

Back to the data listed in Table 3.1. The third design has the smallest mean 2.04 and variance 0.58. The mean of the sixth design is 7.31 which is 3.6 times of the mean of the third design. If the FR distribution for the sixth design could be scaled down to 1/3.6 of its current distribution, the variance would be $2.02/3.6^2 = 0.159$, which is smaller than 0.58. If the same scaled back operation were performed to the rest of the sample distributions, the third design would have the largest variance, as shown in the last column of Table 3.1. Moreover, the ordering of SN ratios matches exactly with the ordering of the rescaled variances. This is not a coincidence. The sample means can be scaled to any common number, the ordering of rescaled variances will remain the same, though their sizes may be different. This fact shows that SN ratios correctly reflect the relative consistency of the competing FR distributions.

4 Conclusions and Reconciliation

Before the process of design improvement can begin, it is important to select an appropriate criterion which defines the objective and measures the progress in design iterations. Both Suh and Taguchi concurred in their use of the reproducibility probability or its algorithm, the information, as the measure of design merit. They use, however, two different data strategies to maximize the reproducibility. Suh is inclined to increase it directly; Taguchi first partitions the data, then reduces the bias and the intrinsic dispersion separately.

Due to the dimensionless and scale-consistent property of reproducibility, it can be used to compare FR distributions of different dimensions, means, and scales. Furthermore, the multiplicative rule provides a straightforward way to combine multiple probabilities into a single measure of design soundness when the design has several independent FRs, as shown by Suh (1990 P.153). Given the above advantages of reproducibility, it also has several shortcomings. First of all, in order to compute the reproducibility, the FR distribution must be known. In the trial-and-error stage, there is little data about FR distribution. The data obtained from the experimental designs are usually too limited to estimate the FR distribution accurately.

Even when the FR distributions are known, there are other practical inconveniences. Since reproducibility is the proportion of FR which falls into the range of design specifications and since that range is fairly short, the probability becomes zero in many cases. It is especially the case when the FR distribution is biased. Design engineers may judge the experimental design with zero reproducibility a total failure and, hence, discard the associated data completely, even though a failed design may still contain valuable knowledge for design improvement. When this happens, reproducibility itself is not an efficient way to use data.

Since bias is the dominant factor causing zero reproducibility, one way to circumvent zero probability is to separate the mean from the data. After the separation, the bias and the consistency of the FR distribution can be improved by two, hopefully independent, groups of DPs. This strategy naturally calls for a prudent partition of the data.

A measure of dispersion useful to design improvement must be able to show the relative merits of all competing designs. The prerequisite of such property is that the measure should maintain comparableness even when the scales of the FR distributions vary vastly. The unscaled vector of intrinsic dispersion removes both the dimension and the scale from the observed date and render compatibility and comparability. The data and discussions in Section 3 shows that, with good likelihood, the conventional measure of dispersion s^2 will lead to the design with the smallest scale or size, which may not be the correct scale, good for bias minimization.

Taguchi's SN ratio is equivalent to the measure of intrinsic dispersion when the convex function is a quadratic function. His two-step procedure is to maximize SN ratio with one set of DPs and to minimize bias with another set of DPs. With this understanding, it is concluded that Taguchi's prescription is data-efficient for design iteration.

Many product/process design tasks are divided into:

stage 1: partition the design into several sub tasks;

stage 2: design iterations for every sub task;

stage 3: design validations once the above iterations finalize the designs; and

stage 4: predict the reliability of the final product/process design, which is composed of all the validated sub tasks.

In many organizations, the above divisions are handled by different teams/departments.

Suh's independence axiom can be used to guide the partition of design task. For design iteration, statistical design of experiments (DOE) are employed to improve the efficiency, because it systematically configures combinations of DPs. In this stage, targeting is not a matter of primary concern and the FR distribution is likely to be biased. SN ratio or other measure of intrinsic dispersion is more effective to compare relative design merits. In the validation stage, the detailed FR distribution is formulated via physical experiments or computer simulations. Reproducibility can be computed correctly once the *unbiased* FR distribution is available. Suh's prescription is most appropriate in the last stage of design task. Information of sub tasks can be added to form a single measure of design soundness. The final information content can then be used to predict the reliability and to compute the future warranty cost.

Suh's information measures the absolute design merit, whereas Taguchi's SN ratio measures the relative FR consistency. Though robustness is useful in enhancing reproducibility, only information serves as the final judge of design acceptance.

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Distortion Measurements of SVGA PC Monitor

design	y_1	y_2	y_3	y_4	y_5	y_6	y_7	$\bar{y}$	s^2	SN ratio	rescaled s^2
1	5.6	6.7	10.1	4.4	12.3	10.0	9.0	8.30	7.95	9.38	0.48
2	3.3	4.5	4.5	3.3	7.8	5.5	4.5	4.77	2.38	9.82	0.44
3	2.2	1.1	2.2	3.3	2.2	2.2	1.1	2.04	0.58	8.60	0.58
4	2.2	2.2	3.3	4.5	3.3	2.2	2.2	2.84	0.80	10.03	0.42
5	2.2	2.2	3.3	4.5	3.3	3.3	3.4	3.17	0.62	12.06	0.26
6	5.5	6.7	10.0	7.8	6.7	7.8	6.7	7.31	2.02	14.23	0.16
7	2.3	2.2	4.5	4.4	4.5	4.5	4.5	3.83	1.24	10.73	0.35
8	3.3	3.4	4.5	2.2	5.6	5.5	3.3	3.97	1.61	9.92	0.43
9	5.6	5.6	7.8	7.8	7.8	5.6	4.5	6.39	1.90	13.31	0.19

Table 3.1