

$$5.4 \quad \text{Present value of } t\text{-year annuity} = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

$$5.5 \quad \text{Present value of annuity due} = (1+r) \times \text{present value of annuity}$$

$$5.6 \quad \begin{aligned} \text{Future value (FV) of annuity of } \$1 \text{ a year} &= \text{present value of annuity} \\ &\text{of } \$1 \text{ a year} \times (1+r)^t \\ &= \left[ \frac{1}{r} - \frac{1}{r(1+r)^t} \right] \times (1+r)^t = \frac{(1+r)^t - 1}{r} \end{aligned}$$

$$5.7 \quad \text{Future value of annuity due} = \text{future value of ordinary annuity} \times (1+r)$$

$$5.8 \quad 1 + \text{real interest rate} = \frac{1 + \text{nominal interest rate}}{1 + \text{inflation rate}}$$

$$5.9 \quad \text{Real interest rate} \approx \text{nominal interest rate} - \text{inflation rate}$$

## QUESTIONS

### QUIZ

- Present Values.** Compute the present value of a \$100 cash flow for the following combinations of discount rates and times: (LO2)
  - $r = 8\%$ ,  $t = 10$  years.
  - $r = 8\%$ ,  $t = 20$  years.
  - $r = 4\%$ ,  $t = 10$  years.
  - $r = 4\%$ ,  $t = 20$  years.
- Future Values.** Compute the future value of a \$100 cash flow for the same combinations of rates and times as in Quiz Question 1. (LO1)
- Future Values.** In 1880 five aboriginal trackers were each promised the equivalent of 100 Australian dollars for helping to capture the notorious outlaw Ned Kelley. In 1993 the granddaughters of two of the trackers claimed that this reward had not been paid. The Victorian prime minister stated that if this was true, the government would be happy to pay the \$100. However, the granddaughters also claimed that they were entitled to compound interest. How much was each entitled to if the interest rate was 4%? What if it was 8%? (LO1)
- Future Values.** You deposit \$1,000 in your bank account. If the bank pays 4% simple interest, how much will you accumulate in your account after 10 years? What if the bank pays compound interest? How much of your earnings will be interest on interest? (LO1)
- Present Values.** You will require \$700 in 5 years. If you earn 5% interest on your funds, how much will you need to invest today in order to reach your savings goal? (LO2)
- Calculating Interest Rate.** Find the interest rate implied by the following combinations of present and future values: (LO4)

Present Value	Years	Future Value
\$400	11	\$684
183	4	249
300	7	300

- Present Values.** Would you rather receive \$1,000 a year for 10 years or \$800 a year for 15 years if
  - the interest rate is 5%? (LO3)
  - the interest rate is 20%? (LO3)
 Why do your answers to (a) and (b) differ?

8. **Calculating Interest Rate.** Find the annual interest rate. (LO4)

Present Value	Future Value	Time Period
\$100	\$115.76	3 years
200	262.16	4
100	110.41	5

9. **Present Values.** What is the present value of the following cash-flow stream if the interest rate is 6%? (LO3)

Year	Cash Flow
1	\$200
2	400
3	300

10. **Number of Periods.** How long will it take for \$400 to grow to \$1,000 at the interest rate specified? (LO1)
- 4%
  - 8%
  - 16%

11. **Calculating Interest Rate.** Find the effective annual interest rate for each case. (LO5)

APR	Compounding Period
12%	1 month
8	3
10	6

12. **Calculating Interest Rate.** Find the APR (the stated interest rate) for each case. (LO5)

Effective Annual Interest Rate	Compounding Period
10.00%	1 month
6.09	6
8.24	3

13. **Growth of Funds.** If you earn 6% per year on your bank account, how long will it take an account with \$100 to double to \$200? (LO1)
14. **Comparing Interest Rates.** Suppose you can borrow money at 8.6% per year (APR) compounded semiannually or 8.4% per year (APR) compounded monthly. Which is the better deal? (LO5)
15. **Calculating Interest Rate.** Lenny Loanshark charges "1 point" per week (that is, 1% per week) on his loans. What APR must he report to consumers? Assume exactly 52 weeks in a year. What is the effective annual rate? (LO5)
16. **Compound Interest.** Investments in the stock market have increased at an average compound rate of about 5% since 1900. It is now 2007.
- If you invested \$1,000 in the stock market in 1900, how much would that investment be worth today? (LO1)
  - If your investment in 1900 has grown to \$1 million, how much did you invest in 1900? (LO2)
17. **Compound Interest.** Old Time Savings Bank pays 4% interest on its savings accounts. If you deposit \$1,000 in the bank and leave it there, how much interest will you earn in the first year? The second year? The tenth year? (LO1)
18. **Compound Interest.** New Savings Bank pays 4% interest on its deposits. If you deposit \$1,000 in the bank and leave it there, will it take more or less than 25 years for your money to double? You should be able to answer this without a calculator or interest rate tables. (LO1)

19. **Calculating Interest Rate.** A zero-coupon bond that will pay \$1,000 in 10 years is selling today for \$422.41. What interest rate does the bond offer? (LO4)
20. **Present Values.** A famous quarterback just signed a \$15 million contract providing \$3 million a year for 5 years. A less famous receiver signed a \$14 million 5-year contract providing \$4 million now and \$2 million a year for 5 years. Who is better paid? The interest rate is 10%. (LO3)

## PRACTICE PROBLEMS

21. **Compound Growth.** In September 2007 a pound of apples cost \$1.18, while oranges cost \$1.50. Ten years earlier the price of apples was only \$.93 a pound and that of oranges was \$.96 a pound. What was the annual compound rate of growth in the price of the two fruits? If the same rates of growth persist in the future, what will be the price of apples in 2027? What about the price of oranges? (LO6)
22. **Loan Payments.** If you take out an \$8,000 car loan that calls for 48 monthly payments at an APR of 10%, what is your monthly payment? What is the effective annual interest rate on the loan? (LO5)
23. **Annuity Values.** (LO3)
- What is the present value of a 3-year annuity of \$100 if the discount rate is 6%?
  - What is the present value of the annuity in (a) if you have to wait 2 years instead of 1 year for the first payment?
24. **Annuities and Interest Rates.** Professor's Annuity Corp. offers a lifetime annuity to retiring professors. For a payment of \$80,000 at age 65, the firm will pay the retiring professor \$600 a month until death.
- If the professor's remaining life expectancy is 20 years, what is the monthly rate on this annuity? What is the effective annual rate? (LO4)
  - If the monthly interest rate is .5%, what monthly annuity payment can the firm offer to the retiring professor? (LO3)
25. **Annuity Values.** You want to buy a new car, but you can make an initial payment of only \$2,000 and can afford monthly payments of at most \$400. (LO3)
- If the APR on auto loans is 12% and you finance the purchase over 48 months, what is the maximum price you can pay for the car?
  - How much can you afford if you finance the purchase over 60 months?
26. **Calculating Interest Rate.** In a *discount interest loan*, you pay the interest payment up front. For example, if a 1-year loan is stated as \$10,000 and the interest rate is 10%, the borrower "pays"  $.10 \times \$10,000 = \$1,000$  immediately, thereby receiving net funds of \$9,000 and repaying \$10,000 in a year. (LO5)
- What is the effective interest rate on this loan?
  - If you call the discount  $d$  (for example,  $d = 10\%$  using our numbers), express the effective annual rate on the loan as a function of  $d$ .
  - Why is the effective annual rate always greater than the stated rate  $d$ ?
27. **Annuity Due.** Recall that an annuity due is like an ordinary annuity except that the first payment is made immediately instead of at the end of the first period. (LO3)
- Why is the present value of an annuity due equal to  $(1 + r)$  times the present value of an ordinary annuity?
  - Why is the future value of an annuity due equal to  $(1 + r)$  times the future value of an ordinary annuity?
28. **Rate on a Loan.** If you take out an \$8,000 car loan that calls for 48 monthly payments of \$240 each, what is the APR of the loan? What is the effective annual interest rate on the loan? (LO5)
29. **Loan Payments.** Reconsider the car loan in the previous question. What if the payments are made in four annual year-end installments? What annual payment would have the same present value as the monthly payment you calculated? Use the same effective annual interest rate as in the previous question. Why is your answer not simply 12 times the monthly payment? (LO5)

30. **Annuity Value.** Your landscaping company can lease a truck for \$8,000 a year (paid at year-end) for 6 years. It can instead buy the truck for \$40,000. The truck will be valueless after 6 years. If the interest rate your company can earn on its funds is 7%, is it cheaper to buy or lease? (LO3)
31. **Annuity-Due Value.** Reconsider the previous problem. What if the lease payments are an annuity due, so that the first payment comes immediately? Is it cheaper to buy or lease? (LO3)
32. **Annuity Due.** A store offers two payment plans. Under the installment plan, you pay 25% down and 25% of the purchase price in each of the next 3 years. If you pay the entire bill immediately, you can take a 10% discount from the purchase price. Which is a better deal if you can borrow or lend funds at a 5% interest rate? (LO3)
33. **Annuity Value.** Reconsider the previous question. How will your answer change if the payments on the 4-year installment plan do not start for a full year? (LO3)
34. **Annuity and Annuity-Due Payments.** (LO3)
- If you borrow \$1,000 and agree to repay the loan in five equal annual payments at an interest rate of 12%, what will your payment be?
  - What if you make the first payment on the loan immediately instead of at the end of the first year?
35. **Valuing Delayed Annuities.** Suppose that you will receive annual payments of \$10,000 for a period of 10 years. The first payment will be made 4 years from now. If the interest rate is 5%, what is the present value of this stream of payments? (LO3)
36. **Mortgage with Points.** Home loans typically involve “points,” which are fees charged by the lender. Each point charged means that the borrower must pay 1% of the loan amount as a fee. For example, if the loan is for \$100,000 and 2 points are charged, the loan repayment schedule is calculated on a \$100,000 loan but the net amount the borrower receives is only \$98,000. What is the effective annual interest rate charged on such a loan assuming loan repayment occurs over 360 months? Assume the interest rate is 1% per month. (LO3)
37. **Amortizing Loan.** You take out a 30-year \$100,000 mortgage loan with an APR of 6% and monthly payments. In 12 years you decide to sell your house and pay off the mortgage. What is the principal balance on the loan? (LO3)
38. **Amortizing Loan.** Consider a 4-year amortizing loan. You borrow \$1,000 initially, and repay it in four equal annual year-end payments. (LO3)
- If the interest rate is 8%, show that the annual payment is \$301.92.
  - Fill in the following table, which shows how much of each payment is interest versus principal repayment (that is, amortization), and the outstanding balance on the loan at each date.

Time	Loan Balance	Year-End Interest Due on Balance	Year-End Payment	Amortization of Loan
0	\$1,000	\$80	\$301.92	\$221.92
1	_____	_____	301.92	_____
2	_____	_____	301.92	_____
3	_____	_____	301.92	_____
4	0	0	—	—

- Show that the loan balance after 1 year is equal to the year-end payment of \$301.92 times the 3-year annuity factor.
39. **Annuity Value.** You’ve borrowed \$4,248.68 and agreed to pay back the loan with monthly payments of \$200. If the interest rate is 12% stated as an APR, how long will it take you to pay back the loan? What is the effective annual rate on the loan? (LO3)
40. **Annuity Value.** The \$40 million lottery payment that you just won actually pays \$2 million per year for 20 years. If the discount rate is 8% and the first payment comes in 1 year, what is the present value of the winnings? What if the first payment comes immediately? (LO3)
41. **Real Annuities.** A retiree wants level consumption in real terms over a 30-year retirement. If the inflation rate equals the interest rate she earns on her \$450,000 of savings, how much can she spend in real terms each year over the rest of her life? (LO6)

42. **EAR versus APR.** You invest \$1,000 at a 6% annual interest rate, stated as an APR. Interest is compounded monthly. How much will you have in 1 year? In 1.5 years? (LO5)
43. **Annuity Value.** You just borrowed \$100,000 to buy a condo. You will repay the loan in equal monthly payments of \$804.62 over the next 30 years. What monthly interest rate are you paying on the loan? What is the effective annual rate on that loan? What rate is the lender more likely to quote on the loan? (LO3)
44. **EAR.** If a bank pays 6% interest with continuous compounding, what is the effective annual rate? (LO5)
45. **Annuity Values.** You can buy a car that is advertised for \$24,000 on the following terms: (a) pay \$24,000 and receive a \$2,000 rebate from the manufacturer; (b) pay \$500 a month for 4 years for total payments of \$24,000, implying zero percent financing. Which is the better deal if the interest rate is 1% per month? (LO3)
46. **Continuous Compounding.** How much will \$100 grow to if invested at a continuously compounded interest rate of 10% for 8 years? What if it is invested for 10 years at 8%? (LO5)
47. **Future Values.** I now have \$20,000 in the bank earning interest of .5% per month. I need \$30,000 to make a down payment on a house. I can save an additional \$100 per month. How long will it take me to accumulate the \$30,000? (LO3)
48. **Perpetuities.** A local bank advertises the following deal: "Pay us \$100 a year for 10 years and then we will pay you (or your beneficiaries) \$100 a year forever." Is this a good deal if the interest rate available on other deposits is 6%? (LO3)
49. **Perpetuities.** A local bank will pay you \$100 a year for your lifetime if you deposit \$2,500 in the bank today. If you plan to live forever, what interest rate is the bank paying? (LO4)
50. **Perpetuities.** A property will provide \$10,000 a year forever. If its value is \$125,000, what must be the discount rate? (LO4)
51. **Applying Time Value.** You can buy property today for \$3 million and sell it in 5 years for \$4 million. (You earn no rental income on the property.) (LO3)
- If the interest rate is 8%, what is the present value of the sales price?
  - Is the property investment attractive to you? Why or why not?
  - Would your answer to (b) change if you also could earn \$200,000 per year rent on the property?
52. **Applying Time Value.** A factory costs \$400,000. You forecast that it will produce cash inflows of \$120,000 in year 1, \$180,000 in year 2, and \$300,000 in year 3. The discount rate is 12%. Is the factory a good investment? Explain. (LO3)
53. **Applying Time Value.** You invest \$1,000 today and expect to sell your investment for \$2,000 in 10 years. (LO1)
- Is this a good deal if the discount rate is 6%?
  - What if the discount rate is 10%?
54. **Calculating Interest Rate.** A store will give you a 3% discount on the cost of your purchase if you pay cash today. Otherwise, you will be billed the full price with payment due in 1 month. What is the implicit borrowing rate being paid by customers who choose to defer payment for the month? (LO4)
55. **Quoting Rates.** Banks sometimes quote interest rates in the form of "add-on interest." In this case, if a 1-year loan is quoted with a 20% interest rate and you borrow \$1,000, then you pay back \$1,200. But you make these payments in monthly installments of \$100 each. What are the true APR and effective annual rate on this loan? Why should you have known that the true rates must be greater than 20% even before doing any calculations? (LO5)
56. **Compound Interest.** Suppose you take out a \$1,000, 3-year loan using add-on interest (see previous problem) with a quoted interest rate of 20% per year. What will your monthly payments be? (Total payments are  $\$1,000 + \$1,000 \times .20 \times 3 = \$1,600$ .) What are the true APR and effective annual rate on this loan? Are they the same as in the previous problem? (LO5)
57. **Calculating Interest Rate.** What is the effective annual rate on a 1-year loan with an interest rate quoted on a discount basis (see Practice Problem 26) of 20%? (LO4)

58. **Effective Rates.** First National Bank pays 6.2% interest compounded semiannually. Second National Bank pays 6% interest, compounded monthly. Which bank offers the higher effective annual rate? (LO5)
59. **Calculating Interest Rate.** You borrow \$1,000 from the bank and agree to repay the loan over the next year in 12 equal monthly payments of \$90. However, the bank also charges you a loan-initiation fee of \$20, which is taken out of the initial proceeds of the loan. What is the effective annual interest rate on the loan taking account of the impact of the initiation fee? (LO4)
60. **Retirement Savings.** You believe you will need to have saved \$500,000 by the time you retire in 40 years in order to live comfortably. If the interest rate is 6% per year, how much must you save each year to meet your retirement goal? (LO3)
61. **Retirement Savings.** How much would you need in the previous problem if you believe that you will inherit \$100,000 in 10 years? (LO3)
62. **Retirement Savings.** You believe you will spend \$40,000 a year for 20 years once you retire in 40 years. If the interest rate is 6% per year, how much must you save each year until retirement to meet your retirement goal? (LO3)
63. **Retirement Planning.** A couple thinking about retirement decide to put aside \$3,000 each year in a savings plan that earns 8% interest. In 5 years they will receive a gift of \$10,000 that also can be invested. (LO3)
- How much money will they have accumulated 30 years from now?
  - If their goal is to retire with \$800,000 of savings, how much extra do they need to save every year?
64. **Retirement Planning.** A couple will retire in 50 years; they plan to spend about \$30,000 a year in retirement, which should last about 25 years. They believe that they can earn 8% interest on retirement savings. (LO3)
- If they make annual payments into a savings plan, how much will they need to save each year? Assume the first payment comes in 1 year.
  - How would the answer to part (a) change if the couple also realize that in 20 years, they will need to spend \$60,000 on their child's college education?
65. **Real versus Nominal Dollars.** An engineer in 1950 was earning \$6,000 a year. Today she earns \$60,000 a year. However, on average, goods today cost 6.9 times what they did in 1950. What is her real income today in terms of constant 1950 dollars? (LO6)
66. **Real versus Nominal Rates.** If investors are to earn a 3% real interest rate, what nominal interest rate must they earn if the inflation rate is
- zero? (LO6)
  - 4%? (LO6)
  - 6%? (LO6)
67. **Real Rates.** If investors receive a 6% interest rate on their bank deposits, what real interest rate will they earn if the inflation rate over the year is
- zero? (LO6)
  - 3%? (LO6)
  - 6%? (LO6)
68. **Real versus Nominal Rates.** You will receive \$100 from a savings bond in 3 years. The nominal interest rate is 8%.
- What is the present value of the proceeds from the bond? (LO2)
  - If the inflation rate over the next few years is expected to be 3%, what will the real value of the \$100 payoff be in terms of today's dollars? (LO6)
  - What is the real interest rate? (LO6)
  - Show that the real payoff from the bond [from part (b)] discounted at the real interest rate [from part (c)] gives the same present value for the bond as you found in part (a). (LO6)
69. **Real versus Nominal Dollars.** Your consulting firm will produce cash flows of \$100,000 this year, and you expect cash flow to keep pace with any increase in the general level of prices. The interest rate currently is 6%, and you anticipate inflation of about 2%.
- What is the present value of your firm's cash flows for years 1 through 5? (LO6)
  - How would your answer to (a) change if you anticipated no growth in cash flow? (LO2)