CS 171
Homework Assignment 9h

Given: March 22, 2012 **Due:** March 27, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

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1. Prove that for $n \in \mathbb{N}$, with $n \geq 2$, define s_n by

$$s_n = \left(1 - \frac{1}{2}\right) \times \left(1 - \frac{1}{3}\right) \times \cdots \times \left(1 - \frac{1}{n}\right).$$

Prove that $s_n = 1/n$ for every natural number $n \geq 2$.

2. Prove that $\sqrt{6}$ is irrational.

3. Prove the following using induction.

(a)$\forall n \geq 1, \sum_{j=1}^{n} j^2 = n(n+1)(2n+1)/6$.
(b) If n is a positive integer and $1 + x > 0$ then $(1 + x)^n \geq 1 + nx$.
(c) for all positive integers n and for all integers $x > 1$, $x^n - 1$ is divisible by $x - 1$.

4. Find the mistake in the attempted proof given below of the following ridiculous claim: "All horses are of the same color".

Let $P(n)$ be the property that n horses are of the same color.
Base Case: $P(1)$ is clearly true since there is only one horse.
Induction hypothesis: Assume that $P(k)$ is true for some $k > 0$.
Induction Step: We want to prove that $P(k + 1)$ is true. Consider any set of $k + 1$ horses and number them $1, 2, \ldots, k+1$. By induction hypothesis, the first k horses are of the same color. Also, the last k horses are of the same color, again by induction hypothesis. Since the set of first k horses and the set of last k horses overlap, all $k + 1$ horses must be of the same color. Thus, $P(k + 1)$ is true. This completes the proof.

5. Suppose that m and n are integers such that $n^2 + 1 = 2m$. Prove that m is the sum of the squares of two non-negative integers.