CS 171
Homework Assignment 7h

Given: March 01, 2012 **Due:** March 06, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

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1. Consider a biased coin that has a probability of 2/3 of landing heads, probability of 1/4 of landing tails, and probability of 1/12 of landing on its edge. We flip the coin n times.
(a) What is the probability that at least three tails occur?
(b) What is the expected number of times that it lands on its edge?

2. Give an example that shows that the variance of the sum of two random variables is not necessarily equal to the sum of their variances when the random variables are not independent.

3. A player rolls a die. They stop at the fourth roll or when a six appears, whichever occurs first. Let X be the number of rolls. What is $\mathbf{E}[X]$ and $\mathrm{Var}[X]$?

4. A total of 4 groups having 4 teachers (one for each group) and 148 kids from the same preschool arrive at the aquarium. The groups have, respectively, 40, 33, 25, and 50 kids. One of the kids is randomly selected. Let X denote the number of kids that were in the group carrying this randomly selected kid. One of the 4 teachers is also randomly selected. Let Y denote the number of kids in this randomly selected teacher's group. Compute $\mathbf{E}[X], \mathbf{E}[Y], \mathrm{Var}[X], \mathrm{Var}[Y]$.

5. An airline knows that on average 5% of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can only hold 50 passengers. What is the probability that there will be a seat available for every passenger that shows up? Your answer must not involve the summation symbol.

6. Consider the following bad algorithm for sorting an array A of n different integers:

1. Randomly shuffle A.
2. For $i \leftarrow 0$ to $n-2$, do the following:
3. If $A[i] > A[i+1]$, go back to step 1 and start over again.

(a) Let X be the total number of times the algorithm executes step 1. Compute $\mathbf{E}[X]$. Your answer should be a simple function of n. Show your work.
(b) Suppose $n = 3$ and let Y be the total number of times the algorithm executes the comparison in step 3. Calculate $\mathbf{E}[Y]$.