CS 171
Homework Assignment 6h

Given: February 23, 2012 **Due:** February 28, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

---

1. An integer is chosen uniformly at random from the range $[1, 100000]$. What is the probability that the number is divisible by one or more of 4, 6, 9?

2. There are two balls. Each of the balls is painted either black or gold and then placed in an urn. Suppose that each ball is colored black with probability 1/2, and that these events are independent.
(a) Suppose that you obtain information that gold paint has been used (and thus at least one of the balls is painted gold). Compute the conditional probability that both balls are painted gold.
(b) Suppose, now, that the urn tips over and one ball falls out. It is painted gold. What is the probability that both balls are gold in this case?

3. After grading the final exam the instructor decides to pass two out of the three students in CS 171. One of the students asks the instructor to tell him the identity of a student other than himself who will pass. The instructor refuses with the following rationale: at your present state of knowledge, your probability of passing the course is 2/3, but after you know my answer, your probability of passing the course will become 1/2, since there will be two students (including yourself) whose fate is unknown and exactly one of the two will be pass. Is the instructor's reasoning correct? Justify your answer.

4. Suppose we roll a fair k-sided die with the numbers 1 through k on the die's faces. If X is the number that appears, what is $\mathbf{E}[X]$?

5. A carnival game consists of three dice in a box. A player can bet a dollar on any of the numbers 1 through 6. The box is shaken, and the payoff is as follows. If the player's number doesn't appear on any of the dice, he loses his dollar. Otherwise if his number appears on exactly k of the three dice, $k = 1, 2, 3$, he keeps his dollar and wins k more dollars. What is his expected gain from playing the carnival game once?

6. Suppose we roll six fair, independent dice. Let R be the *largest* number that comes up. Compute $\mathbf{E}[R]$.