CS 171
Homework Assignment 11h

Given: April 05, 2012 **Due:** April 10, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

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1. Prove that a connected graph with n vertices has exactly one cycle if and only if it has exactly n edges.

2. Suppose that every vertex of a graph has degree at least 3. Prove that G has a cycle of even length. (*Hint:* Consider a maximal path).

3. There are n cities in a OneWayCountry. Every pair of cities is connected by exactly one direct road. Show that there exists a city which can be reached from every other city either directly or via at most one other city. (*Hint:* Use induction.)

4. Prove that a graph with maximum degree k is $(k+1)$-colorable.

5. Let T_1 and T_2 be spanning trees of a graph G and let e be an edge of T_1 not in T_2. Show that there is a spanning tree T_3 of G containing e and all but one edges from T_2.