CS 171
Homework Assignment 10h

Given: March 29, 2012 **Due:** April 03, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

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1. Prove, using induction, that every positive integer can be expressed as a sum of distinct powers of 2. For example, $13 = 2^3 + 2^2 + 2^0$.

2. Prove the following using induction.

(a) For all positive integers n, and for any integers a and b with $a \neq b$, $a^n - b^n$ is divisible by $a - b$.
(b) For all positive integers n,

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} < 2\sqrt{n}$$

3. Suppose you take a piece of paper and draw a bunch of straight lines, no one exactly on top of another, that completely cross the paper. This divides the paper up into polygonal regions. Prove by induction that you can always color the various regions using only *two* colors, so that any two regions that share a boundary line have different colors. Regions that share only a boundary point are permitted to have the same color.

4. Prove or disprove the following. In any group of two or more people, there are always at least two people who have the same number of friends. Assume that if a person p is a friend of a person q then q is also a friend of p.