So Formula 6 gives

$$\begin{aligned}\int_0^1 \tan^{-1} x\,dx &= x\tan^{-1} x\Big]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx \\ &= 1\cdot\tan^{-1} 0 - 0\cdot\tan^{-1} 1 - \int_0^1 \frac{x}{1+x^2}\,dx \\ &= \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2}\,dx.\end{aligned}$$

To evaluate this integral we use the substitution $t = 1 + x^2$ (since u has another meaning in this example). Then $dt = 2x\,dx$, so $x\,dx = dt/2$. When $x = 0$, $t = 1$; so

$$\begin{aligned}\int_0^1 \frac{x}{1+x^2}\,dx &= \frac{1}{2}\int_1^2 \frac{dt}{t} = \frac{1}{2}\ln|t|\Big]_1^2 \\ &= \frac{1}{2}(\ln 2 - \ln 1) = \frac{1}{2}\ln 2.\end{aligned}$$

Therefore $$\int_0^1 \tan^{-1}\,dx = \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2}\,dx = \frac{\pi}{4} - \frac{\ln 2}{2}.$$