

So Formula 6 gives

$$\begin{aligned}\int_0^1 \tan^{-1} x \, dx &= x \tan^{-1} x \Big|_0^1 - \int_0^1 \frac{x}{1+x^2} \, dx \\ &= 1 \cdot \tan^{-1} 1 - 0 \cdot \tan^{-1} 0 - \int_0^1 \frac{x}{1+x^2} \, dx \\ &= \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2} \, dx.\end{aligned}$$

To evaluate this integral we use the substitution $t = 1 + x^2$ (since u has another meaning in this example). Then $dt = 2x \, dx$, so $x \, dx = dt/2$. When $x = 0$, $t = 1$; so

$$\begin{aligned}\int_0^1 \frac{x}{1+x^2} \, dx &= \frac{1}{2} \int_1^2 \frac{dt}{t} = \frac{1}{2} \ln |t| \Big|_1^2 \\ &= \frac{1}{2} (\ln 2 - \ln 1) = \frac{1}{2} \ln 2.\end{aligned}$$

Therefore
$$\int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2} \, dx = \frac{\pi}{4} - \frac{\ln 2}{2}.$$