1. Find the general solution of the differential equation

$$\frac{dy}{dt} = 3y(y-1).$$

The general solution should include the equilibrium solutions $y = 0$ and $y = 1$. To find the non-equilibrium solutions, we use separation of variables to write

$$\int \frac{dy}{y(y-1)} = \int 3dt. \qquad (1)$$

We can compute the left-hand side of equation (1) by using the algebraic technique "partial fractions" to write

$$\frac{1}{y(y-1)} = \frac{A}{y-1} + \frac{B}{y} \qquad (2)$$

with variables A and B, which we determine quickly as follows:
(i) To find A we multiply equation (2) by $y-1$ to get

$$\frac{1}{y} = A + \frac{B(y-1)}{y}$$

into which we substitute $y = 1$ to get $A = 1$.
(ii) To find B we multiply equation (2) by y to get

$$\frac{1}{y-1} = \frac{Ay}{y-1} + B$$

into which we substitute $y = 0$ to get $B = -1$, so

$$\frac{1}{y(y-1)} = \frac{1}{y-1} - \frac{1}{y}.$$

It follows that

$$\ln(y-1) - \ln(y) = 3t + C,$$

or equivalently,

$$\ln(1 - 1/y) = 3t + C.$$

Solving for y gives

$$1 - 1/y = Ke^{3t}$$

hence

$$y = \frac{1}{1 - Ke^{3t}}$$