ANSWERS TO EXAM 1 SAMPLE QUESTIONS

1) A; None B: 1 C: None

<u>Explanation</u>. All three equations have equilibrium values $y=0$, $y=2$, $y=-2$, but only slope field #1 has horizontal slopes for $y=0$, $y=2$ and $y=-2$.
It seems that slope field #1 is for an autonomous equation because slopes appear to depend only on height, y.
Therefore, #1 can only match B or C, and B looks closer.

2) $\frac{dy}{dt} = -4y+7 \Rightarrow \int \frac{dy}{-4y+7} = \int dt \Rightarrow$

$$-\frac{1}{4}\ln(7-4y) = -t + C \Rightarrow$$

$$\ln(7-4y) = -4t + C \Rightarrow$$

$$7-4y = ke^{-4t} \Rightarrow$$

$$\boxed{y = -\frac{1}{4}ke^{-4t} + \frac{7}{4}}$$

3) For the family A it is easy to see ~~that~~ $x=0$ is a node. Only #4 could match, so A: 4.

For the family B we have
$\frac{d}{dx}(Ax-x^3) = A-3x^2 = A>0$ for $x=0$, $A>0$, so $x=0$ is a source, so B: 2.

3) $\boxed{A : 4 \text{ and } B : 2}$ Explained above

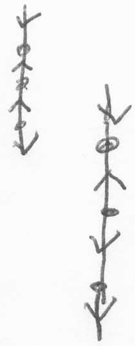
4) $\sin^2(y) = 0 \iff y = \pi n$ for some integer n

Since $\sin^2(y) > 0$ for all other values of y, all of the eq. pts. are nodes.

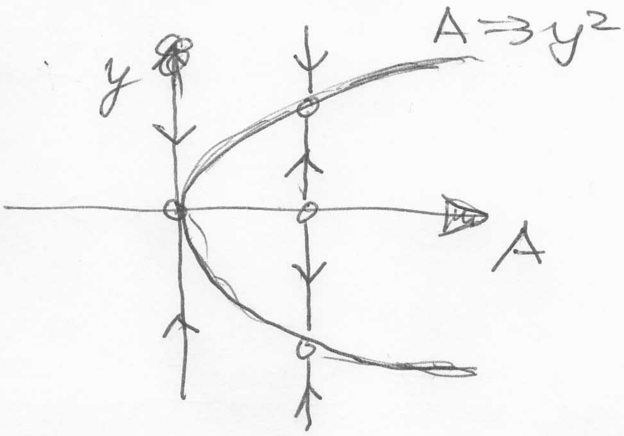
5) 1: A

2: None

3: None



6) $\frac{d}{dy}(y^3 - Ay) = 3y^2 - A$ so $y = \pm\sqrt{\frac{A}{3}}$ for $A \geq 0$



7) $\frac{dy}{dt} = 3y(y-1)$

8) Suppose that $f(t, y)$ and $\frac{\partial f}{\partial y}$ are continuous functions in a rectangle of the form $\{(t, y) : a < t < b,\ c < y < d\}$ in the (t, y)-plane

8) continued

If (t_0, y_0) is a point in this rectangle, then there is an $\varepsilon > 0$ and a function $y(t)$ defined for t with $t_0 - \varepsilon < t < t_0 + \varepsilon$ that solves the initial-value problem

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

If $y_1(t)$ and $y_2(t)$ are two functions that solve the initial value problem for $t_0 - \varepsilon < t < t_0 + \varepsilon$, then

$$y_1(t) = y_2(t) \text{ for } t \text{ such that } t_0 - \varepsilon < t < t_0 + \varepsilon.$$

To exhibit solutions of

$$\frac{dy}{dt} = \frac{y}{t}$$

passing through $(0,0)$, separate variables.

$$\frac{dy}{y} = \frac{dt}{t} \Rightarrow \int \frac{dy}{y} = \int \frac{dt}{t} \Rightarrow$$

$$\ln(y) = \ln(t) + C \Rightarrow y = e^{\ln(t) + C} = Kt.$$

All such solutions pass through $(t, y) = (0,0)$.

This does not contradict the theorem because the condition that $\frac{\partial f}{\partial y}$ be continuous does not hold:

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{t}\right) = \frac{1}{t} \text{ is discontinuous for } t = 0.$$

9. (a) $\frac{dy}{dt} = 2.4 - 4y \Rightarrow$

$\int \frac{dy}{2.4-4y} = \int dt \Rightarrow$

$-\frac{1}{4}\ln(2.4-4y) = t + C \longrightarrow$

$\ln(2.4-4y) = -4t + C \Longrightarrow$

$2.4 - 4y = ke^{-4t} \Longrightarrow$

$y = 0.6 - \frac{k}{4}e^{-4t}$ or

$\boxed{y = 0.6 - Ke^{-4t}} \to \boxed{y = 0.6 - e^{-4t}}$

Check: $\frac{dy}{dt} = 4Ke^{-4t}$

$2.4 - 4y = 2.4 - 2.4 + 4Ke^{-4t}$

(b)

(C) $y = 0$ or $t^2 = 5$

(d) $y(t) = 0$

10. A) $\frac{d}{dy}(1 - y^{1/2}) = \frac{-y^{-1/2}}{2} < 0 \Rightarrow$ sink

B) $\frac{d}{dy}(y^2 + 4y) = (2y+4)\Big|_{y=1} > 0$: source ✓ Yes

C) Yes : $(y-1)^3 < 0$ if $y < 1$, " " > 0 if $y > 1$.

D) No

11. $\frac{dy}{dt} = y^2(t-2)$

$\int \frac{dy}{y^2} = \int (t-2)\,dt \implies$

$-y^{-1} = \frac{1}{2}t^2 - 2t + C \implies$

$\frac{1}{y} = 2t - \frac{1}{2}t^2 + C \implies$

$$\boxed{y = \frac{1}{2t - \frac{1}{2}t^2 + C}}$$

$y(0) = \sqrt{5} \implies \left.\frac{1}{2t - \frac{1}{2}t^2 + C}\right|_{t=0} = \sqrt{5} \implies \frac{1}{C} = \sqrt{5}$

$\implies C = 1/\sqrt{5} \implies$

$$\boxed{y = \frac{1}{2t - \frac{1}{2}t^2 + \frac{1}{\sqrt{5}}}}$$

The general solution does not contain any $y(t)$ such that $y(0) = 0$, but $\boxed{y(t) \equiv 0}$ is a solution for the initial-value problem $y(0) = 0$.

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