This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be the student’s own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

1. Answer the following questions. You don’t need to show your work. Questions a-d are based on the following scenario: Production line I of a factory works 60% of the time, while production line II works 70% of the time, independently of each other. Questions e-i are based on the following scenario. Suppose that each child born to a couple is equally likely to be a boy or a girl independent of the sex distribution of the other children of the family. Suppose the couple has 5 children.

   a. What is the probability that both lines operate?
   b. What is the probability that both lines are stopped?
   c. What is the probability that at least one of the lines operate?
   d. What is the probability that precisely one of the lines operate?
   e. What is the probability that all children are of the same sex?
   f. What is the probability that the 3 eldest are boys and the others are girls?
   g. What is the probability that exactly 3 are boys?
   h. What is the probability that the 2 oldest are girls?
   i. What is the probability that there is at least 1 girl?

2. Die A has 4 red and 2 white faces, whereas die B has 2 red and 4 white faces. A fair coin is flipped once. If it lands on heads, the game continues with die A; if it lands tails, then die B is to be used.

   a. Show that the probability of red at any throw is 1/2.
   b. If the first two throws result in red, what is the probability of red at the third throw?
   c. If red turns up at the first two throws, what is the probability that it is die A that is being used?
3. The chance of a baby surviving delivery is 98%. However, 15% of all births involve C-section, and when a C-section is performed the baby survives 96% of the time. If a randomly chosen pregnant woman does not have a C-section, what is the probability that her baby survives?

4. Between Ahmedabad, Bombay, and Calcutta, there are the following highways: Ahmedabad – Bombay, Ahmedabad – Calcutta, and Bombay – Calcutta. During monsoon, when there is heavy rain, each of the road gets blocked independently with probability $p$. What is then the probability that Calcutta will be accessible from Ahmedabad?

5. Suppose you know that Mr. and Mrs. Vincent have at most eight children. One day you happen to meet two girls who were the Vincent children. When you told the Vincents about this they told you that the chance of this happening is exactly 1/2, i.e., the probability that two of the Vincent children, randomly chosen, are both girls is exactly 1/2. How many children do the Vincent’s have, exactly? Of them what is the exact number of girls?

6. Two equally matched baseball teams play a series of games. The winner of the series is the first team to win three games. What is the probability that the team that wins the first game wins the series?

7. A fair six-sided die is rolled 3 times. Let $X$ be the number of times the number 6 comes up. Compute $\Pr[X = 1|X \geq 1]$?

8. Consider an urn containing one fair coin and one two-headed coin. Suppose that Alice randomly draws a coin from the urn and flips it twice.

   (a) What is the sample space of the experiment?

   (b) Given that both flips produce heads, what is the probability that Alice drew the two-headed coin from the urn?