Formulas.

\[ \lim_{t \to \infty} \arctan(t) = \frac{\pi}{2} \]

\[ \lim_{t \to -\infty} \arctan(t) = -\frac{\pi}{2} \]

1. (a) For which values of \( p \) does the following integral converge? \( \int_1^\infty x^{-p} \, dx \)

(b) For which values of \( p \) does the following integral converge? \( \int_0^1 x^{-p} \, dx \)

2. (a) Compute \( \int_2^\infty x^{-3} \, dx \) if it exists.

   Hint: this is an improper integral, so its ”value” should be expressed as a limit.

(b) Compute \( \int_1^\infty \sin \pi x \, dx \) if it exists.

3. (a) Compute \( \int_0^3 \frac{1}{\sqrt{x}} \, dx \) if it exists.

(b) Compute \( \int_0^3 \frac{1}{x \sqrt{x}} \, dx \) if it exists.

(c) Compute \( \int_1^\infty e^{-2x} \, dx \) if it exists.

(d) Compute \( \int_1^\infty \frac{4x}{1 + x^2} \, dx \) if it exists.

(e) Compute \( \int_{-1}^1 xe^x \, dx \) if it exists.

(f) Compute \( \int_1^\infty \frac{1}{\sqrt{x} + 2} \, dx \) if it exists.

4. For each of the following integrals say whether it is convergent or divergent

(a) \( \int_{-2}^3 \frac{1}{x^4} \, dx \).

(b) \( \int_0^9 \frac{1}{\sqrt{x}} \, dx \).

(c) \( \int_0^1 \frac{\ln x}{\sqrt{x}} \, dx \).

(d) \( \int_0^\pi \sec x \, dx \).

(e) \( \int_0^2 \frac{x - 3}{2x - 3} \, dx \).
5. Define the improper integrals below as limits of ordinary integrals, say whether the integrals are convergent or divergent, and if they are convergent, evaluate the limit.

(a) \[ \int_0^{\infty} \frac{dx}{1 + x^2} \]

(b) \[ \int_0^{\infty} e^{-x} \, dx \]

6. Use integration by parts to evaluate

(a) \[ \int x \cos x \, dx \]
(b) \[ \int xe^x \, dx \]
(c) \[ \int \ln x \, dx \]

7. Evaluate the following trigonometric integrals

(a) \[ \int \sin^2 x \cos x \, dx \]
(b) \[ \int \tan^4 x \sec^4 x \, dx \]
(c) \[ \int \sin^2 x \cos^2 x \, dx \]

8. Write out the form of the partial fraction decomposition. Do not determine the numerical values of the coefficients.

(a) \[ \frac{1}{(3x - 1)(x + 6)^2} \]
(b) \[ \frac{z}{(3z + 5)^3(z + 6)} \]
(c) \[ \frac{x^2 + 1}{(x^2 - 1)} \]
(d) \[ \frac{x}{x^3(x^2 + 1)^2} \]

9. Evaluate the integrals

(a) \[ \int \frac{x^2}{x + 1} \, dx \]
(b) \[ \int \frac{x^2 + 1}{x^2 - x} \, dx \]
(c) \[ \int \frac{2x + 3}{(x + 1)^2} \, dx \]
(d) \[ \int \frac{x^3}{x^2 + 1} \, dx \]
(e) \[ \int \frac{dt}{(t + 4)(t - 1)} \]
10. Define the following improper integrals: (a) \( \int_{a}^{\infty} f(x) \, dx \); (b) \( \int_{-\infty}^{b} f(x) \, dx \); (c) \( \int_{-\infty}^{\infty} f(x) \, dx \)

11. Define the improper integral \( \int_{a}^{b} f(x) \, dx \) in case \( f(x) \) has an infinite discontinuity at \( a \).

12. For each integral below, indicate a technique that can be used to evaluate the integral, and then apply the technique to rewrite the integral as a simpler one. Finally, without actually solving the integral, indicate how to proceed from there.
   (a) \( \int \frac{dx}{\sqrt{9 - x^2}} \)
   (b) \( \int \frac{3x + 2}{(x + 2)(x + 1)(x - 1)} \, dx \)
   (c) \( \int x^3 e^{-x^2} \, dx \)

Choose the best method to use as the first step for evaluating each of the following integrals.

13. See page 537, Example 4
   \( \int \frac{dx}{x \sqrt{\ln x}} \)
   (a) Integration by Parts: \( u = \ldots \) \( dv = \ldots \)
   (b) Substitution: \( u = \ldots \) \( du = \ldots \)
   (c) Inverse Substitution: \( x = \ldots \) \( dx = \ldots \)
   (d) Partial Fractions: Divide First? 
      Decomposition Scheme: 
   (e) Algebraic Simplification: 

14. See page 538, Problem 1
   \( \int \frac{\cos x}{\sqrt{1 + \sin^2 x}} \, dx \)
   (a) Integration by Parts: \( u = \ldots \) \( dv = \ldots \)
   (b) Substitution: \( u = \ldots \) \( du = \ldots \)
   (c) Inverse Substitution: \( x = \ldots \) \( dx = \ldots \)
   (d) Partial Fractions: Divide First? 
      Decomposition Scheme: 
   (e) Algebraic Simplification: 
   (f) Identities (For ln, sin, etc.): 

15. See page 538, Problem 3

\[ \int \frac{e^{\arctan u}}{1 + u^2} \, du \]

(a) Integration by Parts: \( t = \) \( dv = \)

(b) Substitution: \( t = \) \( dt = \)

(c) Inverse Substitution: \( u = \) \( du = \)

(d) Partial Fractions:
   Divide First? ______
   Decomposition Scheme: __________________________

(e) Algebraic Simplification: ______________________________

(f) Identities (For ln, sin, etc.): ______________________________

16. See page 538, Problem 5

\[ \int \sin^2 x \cos^3 x \, dx \]

(a) Integration by Parts: \( u = \) \( dv = \)

(b) Substitution: \( u = \) \( du = \)

(c) Inverse Substitution: \( x = \) \( dx = \)

(d) Partial Fractions:
   Divide First? ______
   Decomposition Scheme: __________________________

(e) Identities (For ln, sin, etc.): ______________________________

17. See page 538, Problem 9

\[ \int \frac{x}{\sqrt{1 - x^2}} \, dx \]

(a) Integration by Parts: \( u = \) \( dv = \)

(b) Substitution: \( u = \) \( du = \)

(c) Inverse Substitution: \( x = \) \( dx = \)

(d) Partial Fractions:
   Divide First? ______
   Decomposition Scheme: __________________________

(e) Algebraic Simplification: ______________________________

(f) Identities (For ln, sin, etc.): ______________________________
18. (10 points) What is the partial fraction decomposition scheme for \( \frac{2x + 3}{(x - 2)(x + 2)} \) ?

(a) \( \frac{Ax}{x - 2} + \frac{B}{x + 2} \)

(b) \( \frac{Ax}{x - 2} + \frac{Bx}{x + 2} \)

(c) \( \frac{A}{x - 2} + \frac{B}{x + 2} \)

(d) \( \frac{Ax + B}{x - 2} + \frac{Cx + D}{x + 2} \).

19. (10 points) To calculate \( J = \int \frac{x^2}{\sqrt{x^2 - 25}} \, dx \), which of the following is the best trigonometric substitution?

(a) \( x = 5 \sin t \)

(b) \( x = 5 \sec t \)

(c) \( x = 5 \tan t \)

(d) None of the above substitutions will allow us to evaluate \( J \).