Exercise II

Remarks: Always explain (or prove if necessary) the correctness of your or claims.

Question 1:

1. How Many different words can we create from ABRACADABRA
2. How many if no two A are one beside the other? Insyructuiion: first arrange the letters that are not A. Then since the A can not be consecutive in creates several places from which you can put A (between the letters I mean).

Question 2: Consider the number of solution to $x_1 + x_2 \ldots + x_k = n$. with non negative integers $x_i$.

1. How many solutions there are under $X_i \geq 0$?
2. How many solutions if every $X_i$ must be either 1 or 2?

Question 3: Consider all sequences of strings of 0, 1 so that there are no two zeroes one after the other. Let $a_n$ be this number.

1. show that $a_n = a_{n-1} + a_{n-2}$. Insuruction:There are two types: a string of length $n$. One starts with 1 so implies no restrictions and one start with 0 and implies a restriction.
2. Show that $a_1 = 2$ and $a_2 = 3$
3. Show that the number of strings with no consecutive ones is the k+2 element in the Fibonacci series $a_1 = F_1 = 1$, $a_2 = F_2 = 1$, $a_4 = F_3 = 2$, $a_4F_4 = 3$, $a_5 = F_5 = 5$ and so on.

Question 4: Show using counting (only) that

$$\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2.$$

Question 5: We throw 5 dice. What is the probability that at least 2 of them has the same value? Instruction: Compute the probability $p$ of the complement event. Then the answer is $1 - p$. 

1