Exercise III

Remarks: In all the answers always explain their correctness (or prove it if necessary).

• **Question 1:** In $B(n, p)$ find the probability of:
  1. No successes
  2. At least one success
  3. The probability of at most one success
  4. At least 2 successes
  5. At least 2 consecutive successes

• **Question 2:**
  1. What is the variance of the random variable that counts the number of 6 that appeared when a fair die is rolled 10 times
  2. Provide an example that shows that the variance of the sum of two random variables may not be equal to the sum of variances

• **Question 3:** Define the Co-variance $Cov(X, Y) = E(X - E(X)) \cdot (Y - E(Y))$
  1. Show that $Cov(X, y) = E(x \cdot Y) - E(X) \cdot E(Y)$
  2. Show that $Cov(X, Y) = 0$ iff $X, Y$ are independent variables.

• **Question 4:** Find $Cov(X, Y)$ if $X$ and $Y$ are the random variables define as follows:
  1. A die is flipped two times. Let $i$ be the random value of the first toss and $j$ of the second
  2. Let $X(i, j) = 2i$
  3. Let $Y(i, j) = i + j$