Not So Cheap Talk:
A Model of Advice with Communication Costs*

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Abstract

We model a game similar to the interaction between an academic advisor and advisee. Like the classic cheap talk setup, an informed player sends information to an uninformed receiver who is to take an action which affects the payoffs of both sender and receiver. However, unlike the classic cheap talk setup, the preferences regarding the receiver's actions are identical for both sender and receiver. Additionally, the sender incurs a communication cost which is increasing in the complexity of the message sent. We characterize the resulting equilibria. Under an additional out-of-equilibrium condition (Condition L), if preferences for sender and receiver are identical then the only equilibria are the most informative, feasible ones. A similar result appears in Chen, Kartik and Sobel (2008) when their No Incentive to Separate (NITS) condition is applied to the case where communication is costless but preferences diverge. Additionally, we model the competency of the advisee by the probability that the action is selected by mistake. We show that the informativeness of the sender is decreasing in the likelihood of the mistake. When the preferences between players diverge and when there are communication costs, we are not guaranteed uniqueness and we provide an example where an increase in communication costs can improve communication.

Preliminary and Incomplete
Comments Welcome

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1 Introduction

Consider the interaction between advisor and advisee in the preparation of a job market paper. The advisor takes a look at the current state of the paper and has a significantly better idea of its shortcomings than does the advisee. Further, the advisor prefers that the advisee correct these shortcomings so that the paper is successful on the job market. The advisor prefers success because either this will reflect well on the advisor or perhaps the advisor might have an intrinsic preference for the success of the advisee. The advisee obviously prefers to correct those shortcomings in order to secure employment, however the nature of the necessary corrections are not known to the advisee. Although there are material incentives for both advisee and advisor to correct the shortcomings, accomplishing this requires the advisor to take time to communicate the nature of these shortcomings. Such communication is costly for the advisor as it would take time out of her busy schedule. Therefore, the advisor decides on the optimal level of detail to communicate to the advisee: more detail increases the quality of the paper but also implies greater communication costs borne by the advisor.

In this paper we analyze the strategic interaction between an informed sender and an uninformed receiver. In our model, the sender learns the state of the world and transmits a message to the receiver. Based on the message, the receiver is to take an action which affects the payoffs of both sender and receiver. We deviate from the classic cheap talk setup in that the sender faces communication costs which are increasing in the complexity of the message.

The sender can send the empty message ($\varnothing$) or can compose a nonempty message from a set of message elements $\{e_1, ...e_\gamma\}$. We assume that the cost of transmitting message m_i is a function of the number of elements in the message. For instance, the cost of transmitting the empty message is $c(0)$ and the cost of transmitting message (e_3, e_5, e_2) is $c(3)$. We view this as the simplest way to model complex communication.

Analogous to the cheap talk literature, equilibrium is partitional: a unique action is induced on connected intervals of the state space. We show that the equilibrium is efficient in that no signal is used in equilibrium when there is a cheaper, unused message. We show that under an out-of-equilibrium condition (Condition L), if preferences for sender and receiver are identical then only the most informative class of equilibria survives. This result is analogous to No Incentive to Seperate ($NITS$) condition when applied to the original cheap talk model. Additionally, we model the competency of the advisee by the probability that the advisee makes a mistake in selecting an action. We show that the informativeness of the sender is decreasing in the probability of a mistake. We interpret this result as suggesting that, in an advising relationship the quality of the advice is increasing in the competency of the advisee. Finally we show that when preferences for sender and receiver are not identical, the Condition L does not guarantee a unique equilibrium and an example where an increase in communication costs improves communication.

2 Related Literature

Despite that every economist has negotiated a relationship with their advisor in graduate school and that many continue to perform the complementary role of advisor, this relationship has garnered relatively little attention in the literature. There are however three related strands of literature, each of which focuses on different issues than we do here. For instance the cheap talk literature examines settings in which communication is costless and the players have different preferences over the action taken. However, we focus on a communication is costly and we allow for the case of identical preferences. Some of the existing costly communication literature focuses on cases where information is either understood or not. However, in our model there can be shades of understanding. The remaining strand of the costly communication literature can exhibit shades of understanding however below we specify their differences with the present paper. Finally, we discuss the empirical literature on the academic advising relationship.

2.1 Cheap Talk and Related Models

The large strand of cheap talk literature was initiated by Crawford and Sobel (1982), hereafter referred to as CS. In the CS model, an informed sender learns the state of the world and decides to communicate some information to an uninformed receiver where the receiver is to take an action which affects the payoffs of both sender and receiver. However, given any state of the world, the sender and receiver have different preferences over the action of the receiver. The authors show that for mild differences in the preferences of receiver and sender, meaningful, albeit incomplete, communication can occur.[1] CS shows that equilibrium always takes the form that the state space is partitioned and the messages are sent such that a unique action is induced within each element of the partition. Our equilibrium is analogous in that a unique message is sent on an interval. We also find that for any nonzero communication costs, the communication is not complete.

A number of papers have extended the original CS model. For instance, Morgan and Stocken (2003) extend the CS model to the case where there is uncertainty regarding the degree of divergence between the preferences of the sender and receiver. Fischer and Stocken (2001) model a situation where the receiver has imperfect information about the state. Blume, Board and Kawamura (2007) modify the CS setup where communication errors (or noise) can occur. In our view, the present paper shares the goal of the above papers: to learn the significance of a particular assumption in the CS model. Here we seek to learn the importance of the assumption that all messages are equally costly (costless) to send irrespective of their complexity.

The original CS model exhibits a large number of possible equilibria. Specifically, CS shows that for a given difference in the preferences of the sender and receiver, if there is an equilibrium where the state space is partitioned into a finite number of partitions (say n) then there are equilibria which partition the state space into 1, 2,.. and $n-1$ elements. Our out-of-equilibrium Condition M leads to a similar result, in that, for a given set of parameter values there exists a maximum number of messages (again say n) which could constitute an equilibria. Additionally under Condition M, there are equilibria where the number of messages equals 1, 2,... and $n-1$.

[1]Spector (2000) shows that in the CS model, as the difference between the preferences of sender and receiver converge, the equilibrium converges towards full information transmition.

As is often the case for multiple equilibria, researchers have sought to reduce the number of cheap talk equilibria through refinements.[2] A recent innovation in this regard is the Condition *No Incentive to Seperate* ($NITS$) as discussed in Chen, Kartik and Sobel (2008). This condition restricts attention to equilibria in which it is not the case that the sender type who receives the lowest possible state ($s = 0$ with a state space of $[0, 1]$) does not prefer to perfectly reveal the state. In their Proposition 3, the authors show that if the monotonicity condition holds in the CS model (as it does in the commonly used "uniform-quadratic" case) then $NITS$ selects a unique equilibrium which is the most informative, i.e. contains the largest possible number of partitions. In our paper, the equilibria under Condition L is similar to $NITS$ in that if an out of equilibrium message is observed then the receiver believes that the state $s = 0$. And similar to $NITS$, Condition L in the case of identical preferences between players, rules out each equilibria except the most informative class of equilibria. However, we also provide an example that when preferences are not identical, Condition L does not guarantee uniqueness. Note that the formal statement of $NITS$ relates to the payoffs of the receiver however implicit in its statement is that, after observing an out-of-equilibrium message, the receiver believes with certainty that $s = 0$. We focus on the beliefs associated with the out-of-equilibrium message, therefore the statement of Condition L specifies beliefs.

Also note that we are not the first to model communication between a sender and receiver who have identical preferences. Morris (2001) presents such a model in which, due to reputation effects, the sender might not truthfully reveal the state of the world.

2.2 Costly Communication

Dewatripont and Tirole (2005) present a communication model where the sender incurs costs of effectively communicating the information and the receiver incurs costs in better absorbing the information. In Dewatripont and Tirole, information is either understood or not. By contrast the states in our model are better characterized by the degree to which they are learned. In Austin-Smith (1994), information acquisition comes at a cost to the sender. Although the receiver cannot verify that the sender is uninformed, the receiver can verify that sender is informed. Austin-Smith shows that the ex-ante uncertainty about the receiver being informed enlarges the set of parameters in which there is an informative equilibrium in the CS model. However, by contrast to our model the sender is completely informed or completely uninformed. In both of the above papers, the sender and receiver have different preferences over the action of the receiver. By contrast, in our paper we consider the case where they are identical.

To our knowledge, there are only two examples of costly communication papers in which there are shades of understanding. In Calvo-Armengol et. al. (2009) the sender transmits a necessarily noisy signal but can affect the precision of the communication by a incurring larger communication cost. In our view this assumption is less appropriate when modeling complex communication as the signal actually sent is not necessarily less complex than the sender's most preferred signal. In Cremer et. al. (2007) a fixed number of partition elements are optimally arranged in order to minimize communication problems between an informed sender and an uninformed receiver who have identical preferences over the action of the receiver.

[2] For instance, see Banks and Sobel (1987), Cho and Kreps (1986), Farrell (1993), Kohlberg and Mertens (1987), Matthews, Okuno-Fujiwara and Postlewaite (1991).

2.3 Empirical Literature

Relevant aspects of our model appear in the academic advising literature. For instance, Knox et. al. (2006) discuss the costs and benefits of being an academic advisor.[3] The benefits of advising include the personal satisfaction involved in guiding a student. Hence, we find support for our assumption that advisor and advisee have identical preferences over the action of the advisee. The costs of advising are primarily composed of the time and energy required by the relationship. Therefore, we regard these as supporting our specification of the payoffs of the advisor.

Schlosser and Kahn (2007) find that advisor and advisee often share the same impression of quality of relationship and of the advisee's competency. We interpret this as confirming the appropriateness of our information assumptions. Additionally, Green and Bauer (1995) find that more capable students receive more supervisory attention than less capable students. Corcoran and Clark (1984) find that more successful researchers received better sponsorship from graduate school advisors than less successful researchers.[4] These findings are in line with Proposition 2, which shows that the informativeness of the sender is decreasing in the probability that the receiver makes a mistake in selecting an action.

3 Model

A sender S and receiver R play a communication game in a single period. Payoffs for both players depend on the receiver's action a, as well as the state of the world s. A state is an element of the closed interval $[0, 1]$. The receiver's action space $\mathcal{A} = [0, 1]$ is equal to the state space. The receiver's utility from action a when the state is s is:

$$u^R(a, s) = -(a - s)^2.$$

The receiver has ex-ante beliefs that the state is uniformly distributed on $[0, 1]$. The sender, observes the state and can communicate some information about the state to S, by sending a message m where $m \in \mathcal{M} = \varnothing \cup \{e_1, ...e_\gamma\}^\infty$ where $\gamma \geq 1$. We interpret message m as more *complex*, and therefore more costly send, than m' if m has more elements than m'. Specifically, communication costs ($c : \mathbb{N} \Rightarrow \mathbb{R}$) are such that $c(i)$ is never less than zero and is an increasing function of the elements in message. We also assume that the empty message is costless, $c(\varnothing) = 0$. Further if message m^i contains i elements and message m^{i+1} contains $i + 1$ elements then we require that $c(i + 1) - c(i) \geq \phi > 0$. The sender's utility is:

$$u^S(a, m^i, s) = -(a - s - b)^2 - c(i)$$

where $b \geq 0$.

The sender's strategy is $\mu : \mathcal{S} \to \mathcal{M}$ and the receiver's strategy is $\alpha : \mathcal{M} \to \mathcal{A}$. We seek an equilibrium where S chooses the optimal action, given beliefs R chooses the optimal action

[3] See Schlosser et. al. (2003) and Schlosser and Gelso (2001) for more on the measurement of the advisee's preferences.

[4] Also see Hollingsworth and Fassinger (2002).

and R's beliefs are derived from Bayes' Rule wherever possible. We denote R's beliefs as $\beta(s|m)$. Specifically, our equilibrium requires that

$$\mu \text{ such that for each } s \in [0,1],\ m \text{ solves } \max_m u^S(a,m,s) \tag{1}$$

$$\alpha \text{ such that for each } m \in M,\ a(m) \text{ solves } \max_a \int u^R(a,s)\beta(s|m)ds \tag{2}$$

and that R's beliefs are derived from S's strategy.

As stated earlier, R uses Bayes' Rule whenever possible, however we have yet to specify the out-of-equilibrium beliefs. We will use one of the following two specifications of out-of-equilibrium beliefs. The first, Condition L, specifies that if an out-of-equilibrium message is observed then R believes that the state is certain to be $s = 0$.

Condition L: Given $(\mu, \alpha; c)$, if there does not exist an $\widehat{s}$ such that $\mu(\widehat{s}) = \widehat{m}$ and R observes $\widehat{m}$ then R believes that S is certain to be $s = 0$.

Although Condition L specifies precise beliefs after observing an out-of-equilibrium message, in the case that $b = 0$, another way to view the condition is that the receiver believes that that state is a particular one among the states with the lowest equilibrium payoffs. In other words, when $b = 0$ there are several states which obtain the lowest equilibrium payoffs and $s = 0$ happens to be a particularly focal one. For a given equilibrium with n messages there will be $n + 1$ states[5] which will satisfy $\arg\max_s (a(m') - s')^2 + c(m')$, so as a matter of convention, R has beliefs that the state is the smallest of these. It is worth pointing out that under Condition L, only a sender with $s = 0$ would send an out-of-equilibrium message if such a sender would prefer to perfectly reveal his state via a costly message.

Condition L is similar to $NITS$ in the sense that the former specifies beliefs in the event of an out-of-equilibrium message which are implicit in the statement of the former. Additionally, for the case that $b > 0$, $s = 0$ is typically not the state with the lowest equilibrium payoffs. So in this sense, we view the reasonableness of Condition L in the case of $b = 0$ to be at least that of $NITS$ in the case that $b > 0$.

The second condition which we consider, Condition M, specifies that if an out-of-equilibrium message is observed then R believes that the state is, among those which induce their optimal action, the one with the largest communication cost.

Condition M: Given $(\mu, \alpha; c)$, if there does not exist an $\widehat{s}$ such that $\mu(\widehat{s}) = \widehat{m}$ and R observes $\widehat{m}$ then R believes with certainty that the state s' with the largest $c(m')$ among those states where $a(m') = s'$.

Condition M supports "more" equilibria and Condition L supports "less." This is because under Condition M an out-of-equilibrium message does not induce an a which is not used in equilibrium and it is therefore relatively difficult to find a deviation from an equilibrium. However, under Condition L an out-of-equilibrium message does induce an a which is not used in equilibrium and so it is relatively easy to find a deviation.

[5] See Lemma 7 in the appendix.

Before we proceed to the results, we briefly discuss some of our modeling choices. First, we designed the model in order to avoid the issue of misrepresentation therefore we can accommodate the case where the message space and state space as distinct ($\mathcal{M} \neq \mathcal{S}$). There exists evidence that in experimental settings, meaningful communication can occur even when there is no a priori meaningful language.[6] Second, the state space is designed to be more rich than the message space.[7] Our state space is uncountably infinite and our message space is countably infinite. In fact, when communication is costly the only equilibria which exist involve a finite number of messages. We believe that this captures an important aspect of reality: it is impossible to completely communicate the complexity of the real world, one may only increase the precision of communication by expending more costly effort.

Modeling communication as we do here has several benefits. First, since we explicitly model the *communication* we feel that yields a certain realism. In our view, modeling complexity is best done with a minimal abstraction of the nature of the complexity otherwise it is tempting to remain in a world where messages are equally costly. Second, we do not have to assume unsophistication on the part of the receiver. For instance, it is possible to imagine communication where the sender incurs a cost which is increasing in the length of the possible interval. However if the sender and receiver are sophisticated, precise communication be be accomplished in a manner in which we view as unsatisfactory. Suppose that the sender wished to communicate the state, $s = 0.315789215$. The sender could send the message leading to the possible interval $[0.317789, 1]$ and the receiver would know that the state is certain to be 0.315789215. To avoid these types of problems, we would either have to model the receiver as unsophisticated or to restrict communication in the way in which we do here. Third, be argue that it should be viewed as more costly to send a message indicating that the state is in the interval, $[0.2, 0.4]$ than in the interval $[0.215789215, 0.415789215]$. Finally, our model make the novel prediction that there is communication through silence for some states of the world.

One potential set of message elements are the set of single digit integers: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Therefore, the sender has the means to truthfully disclose the exact state. However, we will see that, not only will this will not happen in equilibrium, but there are many states for which the sender transmits the empty message.

Although our equilibria share some of the familiar characteristics of the cheap talk literature, the additional results which emerge require the flexibility provided by the notation which we now define. Like the CS equilibria, messages are sent on disjoint intervals. Therefore, we may characterize an equilibrium by a set of cutoff states where we denote the number of messages used in equilibrium as $n + 1$ by listing the order of the messages messages m_0, ..., m_n. The messages induce a set of cutoff states which we denote:

$$0 = s_0 \leq s_1 \leq s_2 \leq ... \leq s_i \leq ... \leq s_n \leq 1 = s_{n+1}. \tag{3}$$

We denote the complexity of the message by a superscript. Therefore message m^i has complexity i and a cost of $c(i)$.

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[6] See Blume et. al. (1998) and Blume et. al. (2001).

[7] This assumption also appears in Lipman (2006).

Equilibrium is such that S's messages are sent as intervals on the state space:[8]

$$\mu(s) = m_i \text{ for } s \in [s_i, s_{i+1}) \tag{4}$$

and R best responds in a straightforward manner:

$$\alpha(m_i) = \overline{a}(s_i, s_{i+1}) = \arg\max_a \int_{s_i}^{s_{i+1}} u^R(a,s)\beta(s|m)ds$$

where $\overline{a}(\underline{s}, \overline{s})$ is the best response of R if the state is known to be between $\underline{s}$ and $\overline{s}$.

The arbitrage equation, also standard in the cheap talk literature, characterizes the equilibrium set of cutoff states:

$$u^S(\overline{a}(s_i, s_{i+1}), m_i, s) = u^S(\overline{a}(s_{i+1}, s_{i+2}), m_{i+1}, s) \text{ for } i \in \{0, ..., n-1\} \tag{A}$$

We define λ_i to be the mass of states such that $\mu(s) = m_i$. Since the messages are sent on an interval of the state space and the states are distributed uniformly, $\lambda_i = s_{i+1} - s_i$ when $\mu(s) = m_i$ for $s \in [s_i, s_{i+1}]$ and $\mu(s) \neq m_i$ for $s \notin [s_i, s_{i+1}]$.

4 Perfect Alignment of Preferences ($b = 0$)

Here we focus on the case where the preferences of sender and receiver are perfectly aligned; in other words, ($b = 0$). In the sequel we analyze the case where there is imperfect alignment of preferences ($b > 0$). Our ultimate goal is to show that only the most informative equilibrium survives under Condition L. We provide the following lemmas to facilitate understanding of the proposition.

Lemma 1 *Consider m^i such that $c(i) = \widehat{c}$. Under Condition L, if $b = 0$ and m^i is used in equilibrium then every m^j where $c(j) \leq \widehat{c}$ is also used in equilibrium.*

Proof: Suppose that there is a equilibrium $(\mu, \alpha; c)$ such that $\mu(s) = m^i$, $c(i) = \widehat{c}$ but no such s' such that $\mu(s') = m^j$ and $c(j) \leq \widehat{c}$. When the signal m^j is observed, R believes that the state is certain to be state 0. A profitable deviation for $s = 0$ is then to send m^j as the communication costs are not greater than that for $(\mu, \alpha; c)$ and the action induced is optimal. Therefore, $(\mu, \alpha; c)$ cannot constitute an equilibrium.
■

The above lemma suggests that each of our equilibria will have no holes: if message m is used in equilibrium then so is every m' which is less than or as costly to send. An implication of Lemma 1 is that there exists states where the sender will be silent by sending the costless, empty message. Another implication of the lemma is that, we can denote an equilibrium by

[8] See the appendix for proof of the results that only one message gets sent for any particular state and the proof of the result that the equilibrium strategy for S entails sending a message m' for states which are conntected intervals.

the most complex message used. Therefore, if the most complex equilibrium messages have k elements then the number of costly equilibrium messages (n) is:

$$n = \sum_{j=1}^{k} \gamma^j$$

If the most costly messages used in equilibrium is k then we say that we have a k-equilibrium.

Definition 1 *A k-**equilibrium** is one in which all messages which cost less than or equal to $c(k)$ are used.*

We will use λ to rewrite expression (A). Consider an k-equilibrium. Message m_i is associated with a mass of $\lambda_i = s_{i+1} - s_i$ for $i \in \{0, ..., n\}$. Additionally, we require that

$$\lambda_i \geq 0 \text{ for every } i \in \{0, ..., n\}$$

and

$$\lambda_0 + \lambda_1 + \lambda_2 + ... + \lambda_n = 1 \tag{5}$$

Lemma 2 *If $b = 0$, the necessary conditions for an n-equilibrium are:*

$$(\lambda^j)^2 - (\lambda^i)^2 = 4\,[c(i) - c(j)]$$

where $n \geq i > j \geq 0$.

Proof: As there are $n+1$ messages used in equilibrium (m_0, m_1, ...,m_n), it must be that there are n equations in expression (A). A typical such expression would be the cutoff state between message m_h^i and m_{h+1}^j where $\mu(s') = m_h^i$ for $s' \in [s_h, s_{h+1})$, $\mu(s'') = m_{h+1}^j$ for $s'' \in [s_{h+1}, s_{h+2})$.

$$-\left(\frac{s_h + s_{h+1}}{2} - s_{h+1}\right)^2 - c(i) = -\left(\frac{s_{h+1} + s_{h+2}}{2} - s_{h+1}\right)^2 - c(j).$$

Without loss of generality, we can write

$$\begin{aligned} -\left(\frac{s_h - s_{h+1}}{2}\right)^2 - c(i) &= -\left(\frac{s_{h+2} - s_{h+1}}{2}\right)^2 - c(j) \\ -\left(\frac{-\lambda^i}{2}\right)^2 &= -\left(\frac{\lambda^j}{2}\right)^2 + c(i) - c(j) \\ (\lambda^j)^2 - (\lambda^i)^2 &= 4\,[c(i) - c(j)] \end{aligned}$$

■

The lemma above establishes the relationship between the mass of states for any two signals for the case of $b = 0$. Only the interval size, not its placement in the state space is related to its communication cost. In other words, the order of the messages does not matter.

We now show that we are guaranteed a maximal k-equilibrium. If a k-equilibrium is to exist then it must be that expression (A) is satisfied for each of the n cutoff states. Given costs $c(0)$, $c(1)$,...,$c(k-1)$, $c(k)$ we can determine whether it is possible, to satisfy expressions (A) and (5). If so, we say that the k-equilibrium is *feasible.*

Lemma 3 *There is always a maximal, feasible k-equilibrium.*

Proof: To check whether a k-equilibrium is feasible we rewrite expressions (A) and (5). In this candidate k-equilibrium there will be

$$n = \sum_{\alpha=1}^{k} \gamma^{\alpha}$$

costly messages. There are γ^k most costly messages which cost $c(k)$. There are γ^{k-1} messages which cost $c(k-1)$. There are γ messages which cost $c(1)$. And there is one message which is costless. The γ^k most costly messages, representing intervals $\lambda_{n-\gamma^k+1}$ through λ_n, which we denote as λ_n. By Lemma 2, γ^{k-1} messages which cost $c(k-1)$, representing intervals $\lambda_{n-\gamma^k-\gamma^{k-1}+1}$ through $\lambda_{n-\gamma^k}$ are sent on an interval of $\sqrt{4c(1)+(\lambda_n)^2}$. The γ^{k-2} messages which cost $c(k-2)$, representing intervals $\lambda_{n-\gamma^k-\gamma^{k-1}-\gamma^{k-2}+1}$ through $\lambda_{n-\gamma^k-\gamma^{k-1}}$ are sent on an interval of $\sqrt{4c(2)+(\lambda_n)^2}$. The γ^2 messages which costs $c(2)$, representing intervals $\lambda_{\gamma+1}$ through $\lambda_{\gamma+\gamma^2}$, are sent on a interval of $\sqrt{4c(k-2)+(\lambda_n)^2}$. The γ messages which cost $c(1)$, representing intervals λ_1 through λ_γ, $\sqrt{4c(k-1)+(\lambda_n)^2}$. Finally for the costless message, we write $\lambda_0 = \sqrt{4c(k)+(\lambda_n)^2}$. Therefore, we may write expression (5) as

$$\sqrt{4c(k)+(\lambda_n)^2} + \gamma\sqrt{4\left[c(k)-c(1)\right]+(\lambda_n)^2} + \gamma^2\sqrt{4\left[c(k)-c(2)\right]+(\lambda_n)^2} + \\ ... + \gamma^{k-2}\sqrt{4\left[c(k)-c(k-2)\right]+(\lambda_n)^2} + \gamma^{k-1}\sqrt{4\left[c(k)-c(k-1)\right]+(\lambda_n)^2} + \gamma^k\lambda_n = 1. \quad (6)$$

Recall that we required that $c(k+1)-c(k) \geq \phi > 0$. Therefore, we can write the lower bound of form each term in the left hand side of expression (6):

$$\sqrt{4k\phi} + \gamma\sqrt{4(k-1)\phi} + \gamma^2\sqrt{4(k-2)\phi} + ... + \gamma^{k-2}\sqrt{4(2)\phi} + \gamma^{k-1}\sqrt{4\phi} > 1. \quad (7)$$

For every γ and ϕ, there is a k large enough so that expression (7) is satisfied. Therefore, we are guaranteed a maximal k-equilibrium.

■

If given c, a k-equilibrium is feasible in a way that $\lambda^n = 0$, we say that k is *exactly feasible.* If given c, a k-equilibrium is feasible in a way that $\lambda^n > 0$, we say that k is *strictly feasible.* Obviously if, given c, expression (6) cannot be satisfied then we will describe a k-equilibrium as not feasible.

Now we are ready for the main result of the section, that the only equilibria under Condition L are the ones in which the most information is transmitted.

Proposition 1 *If $b = 0$ then under Condition L an equilibrium always exists and it is exclusively the most informative, feasible equilibrium.*

Propositioon 1 shows that under Condition L there is only one class of equilibria, only the most complex, feasible equilibria does not have a profitable deviation. Although monotonicity of the equilibrium as found in CS does not hold in our setting, the equilibrium is unique in the sense that in each equilibrium, signals of a given complexity are sent on identical mass. This Proposition is reminiscent of Proposition 3 in Chen, Kartik and Sobel (2008), as they show that in the CS model where monotonicity holds, $NITS$ admits only the most informative equilibrium. In the notation of our model, Chen, Kartik and Sobel show that for $b > 0$ and $c = 0$ in the uniform-quadratic case, NITS uniquely selects the most informative equilibrium. Proposition 1 becomes more surprising when we consider that a subsequent result shows that we are not guaranteed uniqueness when $b > 0$ and $c > 0$.

4.1 Simple Characterization

Here we focus on the case where preferences are perfectly aligned ($b = 0$) and communication costs are linear in the complexity of the message.

For the case of general costs, it is difficult to characterize the threshold level of costs which render a k-equilibrium feasible. However in the linear case, the characterization is rather simple and lends itself to a natural interpretation. If $c(k) \leq c^*(k)$ then a k-equilibrium is feasible and if $c(k) > c^*(k)$ then a k-equilibrium is not feasible. The calculation of $c^*(k)$ is relatively straightforward

Lemma 4 *The cutoff cost for a k-equilibrium is:*

$$c^*(k) = \left(\frac{1}{2\sum_{j=1}^{k} \gamma^{k-j}\sqrt{j}} \right)^2.$$

Proof: At the largest c such that signal k is feasible, it must be that ${\lambda_n}^2 = 0$.
By Lemma 2 it must be that, $\lambda_{n-1}^2 = 4c$, $\lambda_{n-2}^2 = 8c$, ..., $\lambda_1^2 = 4(k-1)c$, $\lambda_0^2 = 4kc$.
Therefore, we may write expression (6) in the case of linear costs as

$$2\sqrt{ck} + 2\gamma\sqrt{c(k-1)} + 2\gamma^2\sqrt{c(k-2)} + ... + 2\gamma^{k-2}\sqrt{c(2)} + 2\gamma^{k-1}\sqrt{c(1)} = 1.$$

and so the lemma is proved.

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Example 1 *Consider the case where $c(i) = 0.01i$ and $\gamma = 1$. Note that:*

$$c^*(4) = 0.00662 < 0.01 < c^*(3) = 0.0145.$$

Under Condition M, there are four classes of equilibria, $n \in \{0, 1, 2, 3\}$. For the $n = 0$ equilibrium, m_0 gets sent for all states. For the $n = 1$ case, there are two equilibria. There is an equilibrium where m_0 is sent for states $[0, 0.52)$ and m_1 for states $[0.52, 1]$. There is another equilibrium where m_1 is sent for states $[0, 0.48)$ and m_0 for states $[0.48, 1]$. Note that in each of the $n = 1$ equilibria $\lambda_0 = 0.52$ and $\lambda_1 = 0.48$. For the $n = 2$ case, there are six equilibria. There is a monotonic equilibria where m_0 is sent for states $[0, 0.392)$, m_1 for states $[0.392, 0.729)$ and m_2 for $[0.729, 1]$. The remaining 5 equilibria require that $\lambda_0 = 0.392$, $\lambda_1 = 0.337$, and $\lambda_2 = 0.271$. For the $n = 3$ case, there are 24 equilibria. There is a monotonic equilibria where m_0 is sent for states $[0, 0.363)$, m_1 for states $[0.363, 0.665)$, m_2

for $[0.665, 0.892)$ *and* m_3 *for* $[0.892, 1]$. *The remaining* 23 *equilibria require that* $\lambda_0 = 0.363$, $\lambda_1 = 0.302$, $\lambda_2 = 0.227$ *and* $\lambda_3 = 0.108$. *For Condition L, only the* 24, $n = 3$ *equilibria are possible.*

Note that we have identified equilibria which the values of λ_i are neither increasing nor decreasing. In other words, Monotonicity (Condition M in CS) does not hold in our setting (unlike the quadratic preferences, uniform state case in CS.) Also since monotonicity fails in this model we should not be surprised that Condition L does not lead to a unique equilibrium as Proposition 1 in Chen, Kartik and Sobel demonstrates that when monotonicity fails in their model, $NITS$ fails to lead to a unique equilibrium.

We are now better equipped to discuss the outcomes if we expand the message space is $\varnothing \cup \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^{\infty}$ where the cost of communication is linear in the number of the elements transmitted, $c(m) = ck$. One might be tempted to conclude that an equilibrium of the following form might exist, $\mu(s) = d$ for $d \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ if $s \in [0.1d, 0.1(d+1))$. However, by Proposition 1 this cannot be an equilibrium because the empty message is not used. Further suppose that $c = 0.1$ then equilibrium in the expanded message space is such that $\lambda_0 = 0.633$ and each single digit is sent on a mass of states $\lambda_i = 0.0366$ for $i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

4.2 Competency of the Advisee

In any advising relationship, the advisor has beliefs regarding the ability of the advisee to execute the advice. It would seem that this competency would influence effort supplied by the advisor. To analyze these issues, we supplement the model to allow for the possibility that the sender might make a mistake in the execution of the action. Specifically, with probability p the receiver perfectly executes the most preferred action:

$$\alpha(m_i) = \overline{a}(s_i, s_{i+1}) = \arg\max_a \int_{s_i^{g(i)}}^{s^{g(i)+1}} u^R(a, s)\beta(s|m)ds.$$

With probability $1-p$, the action a is distributed uniformly on the action space ($U[0,1]$).[9] We interpret p as the competency of the advisee. We now state our result, which demonstrates that the informativeness of the relationship is increasing in the competency of the advisee.

Proposition 2 *Under Condition L the informativeness of the sender is increasing in p*

Proof: In the presence of the possibility of mistakes, the new arbitrage expression can be written:

$$\begin{aligned} & -p\left(\frac{\lambda^j}{2}\right)^2 - (1-p)\int_0^1 (-(x-s)^2)dx - c(i) \\ = \quad & -p\left(\frac{\lambda^i}{2}\right)^2 - (1-p)\int_0^1 (-(y-s)^2)dy - c(j) \end{aligned}$$

[9]Note that we assume that p is unrelated to the message. We could have allowed p to be decreasing in the comlexity of the message, however this would only strenghten our result below.

Therefore, the necessary conditions for equilibrium can be written:

$$(\lambda^j)^2 - (\lambda^i)^2 = \frac{4c}{p}\left[c(i) - c(j)\right].$$

So for any n-equilibrium, a decrease in p will lead to a decrease in $E[(a-s)^2]$.

■

Proposition 2 suggests that in equilibrium S will expend less effort on communication when R exhibits a larger probability of making a mistake in executing the action. We interpret this result as indicating that in equilibrium, advisors will provide more attention to capable advisees. We perform the analysis for $b = 0$ because (as the reader will see) Condition L does not uniquely identify a class of equilibria and therefore comparative statics are problematic

5 Imperfect Alignement of Preferences ($b > 0$)

Recal Proposition 1, that if $b = 0$ and $c > 0$ then under Condition L only the most informative, feasible equilibria exists. This is reminiscent of Proposition 3 of Chen, Kartik and Sobel (2008) which shows that in the quadratic-uniform case, $NITS$ uniquely selects the most informative, feasible equilibria. In the formalism of our paper, the Chen, Kartik and Sobel result corresponds to the case of $b > 0$ and $c = 0$.

We now provide an example that Condition L does not select a unique class of equilibrium when $b > 0$ and $c > 0$. For $b > 0$ and $c > 0$, the lack of uniqueness can manifest itself in two distinct ways. First, there could exist several equilibria with set of equilibrium messages, however these equilibria differ in their informativeness. Second, there can exist equilibria which satisfy Condition L yet differ in the set of equilibrium messages. The following example demonstrates this second aspect and the subsequent example demonstrates the first.

Example 2 *Suppose that $b = 0.245$, $\gamma = 1$ and communication costs are $c(m) = 0.01n$. First, there exists an equilibrium where two messages are used. The costless message is sent on $s \in [0, 0.03)$ and the costly message is sent on $s \in [0.03, 1]$. The sender's $s = 0$ equilibrium payoffs are $-(0.015 - 0.245)^2 = -0.0529$, which is greater than deviation payoffs of $-(0.245)^2 - 0.02 = -0.080$. There also exists an equilibrium with a single equilibrium message. The sender"s $s = 0$ equilibrium payoffs are $-(0.5 - 0.245)^2 = -0.065$, which is greater than deviation payoffs of $-(0.245)^2 - 0.01 = -0.070$.*

This example stands in contrast to the findings of Chan, Kartik and Sobel (2008) who find that $NITS$ uniquely selects only the most informative equilibrium when $b > 0$ and $c = 0$. This also stands in contrast to Proposition 1 in that Condition L uniquely selects only the most informative equilibria when $b = 0$ and $c > 0$.

Note for the case of $b = 0$ and $c > 0$ the order of the messages did not matter as long as their size was governed by Lemma 2. Also for the case of $b > 0$ and $c = 0$ the order of the signals themselves did matter. What did matter in this case was that the order of the intervals were increasing. However when $b > 0$ and $c > 0$ there is an interaction which might cause one of the two forms of the nonuniqueness. It is perhaps not difficult to see that there will exist several equilibria with set of equilibrium messages, however these equilibria differ in their informativeness because the messages have a difference in cost. It is somewhat more difficult to see, (Although Example 2hopefully demonstrates its existence) that there can exist equilibria which satisfy Condition L yet differ in the set of equilibrium messages.

Characterizing the equilibria for the case of $b > 0$ and $c > 0$ is rather difficult. However, we can say something about the orientation of the intervals. It must be that

$$\left(\lambda_{h+1}^{j}\right)^2 - \left(\lambda_h^i\right)^2 = 4b\left[\lambda_{h+1}^{j} + \lambda_h^i\right] + 4\left[c(i) - c(j)\right]$$

Finally, we show that when $b > 0$, there exist equilibria where an increase in communication costs will improve communication. Specifically we show that it is not the case that for all b, an increase in communication costs degrades communication in every equilibria.

Example 3 *When $b = 0.2$, $\gamma = 1$ and $c(m) = 0$, there is only one outcome equivalent equilibria of the following form. A single action is induced on $s \in [0, 0.1)$ and a single action is induced on $s \in [0.1, 1]$. The first message induces $a = 0.05$ and the second induces $a = 0.55$. In this case, $E[-(a-s)^2] = -0.0608$. However when $b = 0.2$, $\gamma = 1$ and $c(m_i) = 0.01i$, there are two non-outcome equivalent equilibria. In the first equilibria, m_0 is sent on $s \in [0, 0.12)$ and m_1 on $s \in [0.12, 1]$. In the second equilibria, m_1 is sent on $s \in [0, 0.08)$ and m_1 on $s \in [0.08, 1]$. In the first equilibria, $E[-(a-s)^2] = -0.0569$ and in the second, $E[-(a-s)^2] = -0.0649$. If the cost of communication is increased, $c(m_i) = 0.02i$ in the first equilibria, m_0 is sent on $s \in [0, 0.14)$ and m_1 on $s \in [0.14, 1]$, implying $E[-(a-s)^2] = -0.0532$.*

The above provides an example where an increase in communication costs can lead to an improvement in communication. Also note that Example 3 contained an instance of two distinct equilibria, which share the set of equilibrium messages yet differ in their informativeness.

6 Conclusions

We have modeled an interaction between an informed sender and uninformed receiver, as in the relationship between academic advisor and advisee. We assume that the advisor faces a cost of communication which is increasing in the complexity of the message sent. We have characterized the equilibria where a unique message is sent on an interval of the state space. When sender and receiver have identical preferences over the action of the receiver, we have demonstrated that under Condition L only the most informative class of equilibria exists. This result is analogous to the application of the No Incentive to Seperate ($NITS$) to the uniform-quadric version of the model in Crawford and Sobel (1982). We have also provided a result which demonstrates that the more competent advisee will enjoy a more informative advising relationship. Finally, for the case that preferences are not identical, we have provided an example where Condition L does not identify a unique equilibrium and that an increase in communication costs might improve communication.

There however remain important questions which are unanswered. For instance, it is not known what happens when sender and receiver have different preferences over action of the receiver. For instance, it is possible to imagine a case where the advisor and advisee have different preferences over the content of the paper. Also, we have modeled the interaction as a single repetition. However, we are interested to learn the equilibrium behavior where the interaction is repeated. There are three possibilities as the relationship is potentially finitely repeated, infinitely repeated or is repeated until the project attains some threshold. An additional issue which arises only in the repeated version of the game relates to learning on the part of the advisee. Presumably there is a relationship between some publicly observable signal and the optimal action for the advisee and also that the advisor wishes to teach the advisee this relationship. Additionally, we are eager to learn the significance of our assumption of quadratic preferences and a uniform probability distribution.

We are currently working on the case where the sender imperfectly observes the state. Our preliminary results suggest that a small amount of noise in this observation improves the quality of communication. This preliminary result is consistent with the findings of Blume et. al. (2007).

Finally, we are eager to learn the validity of the results in an experimental setting. Perhaps the two easiest questions to adress are the following: is communication effort on the part of the sender is not constant over the state space, and do less competent advisees receive less attention. Like most communication games, the equilibria here is quite complicated and this fact makes experimental investigation difficult. On the other hand, experimental papers (for instance Cai and Wang (2006) and Kawagoe and Takizawa (2008)) have found suitable simplifications of the theoretical communication papers which they test. We are confident that a similar such a simplification can be found in our setting.

7 Appendix

Together the below two propositions demonstrate that equilibrium messages are sent only on connected disjoint intervals.

Lemma 5 *For any state $\widehat{s}$, there can only be one message $\widehat{m}$ sent in equilibrium.*

Proof: Suppose that $\mu((s_1, s_2)) = m'$ and $\mu((s_3, s_4)) = m''$ where $(s_1, s_2) \neq (s_3, s_4)$ and $(s_1, s_2) \cap (s_3, s_4) \neq \emptyset$. The sender S can transmit the same information by sending only the least costly of the two, $argmin(c(m'), c(m''))$ and so $\mu((s_4, s_2)) = \{m', m''\}$ cannot be equilibrium.

■

Lemma 6 *The equilibria must be connected intervals.*

Proof: Now suppose there exists m such that $\mu^{-1}(m) = [s_1, s_2) \cup [s_3, s_4)$ with $[s_1, s_2) \neq [s_3, s_4)$, $[s_1, s_2) \cap [s_3, s_4) = \emptyset$ where $s_2 < s_3$.

Case 1: Suppose $\overline{a}(m) \in (s_2, s_3)$. Define $\mu(\overline{a}(m)) = m'$, where $m \neq m'$. If $\overline{a}(m) = \overline{a}(m')$, either $c(m) \neq c(m')$ and there exists a profitable deviation for S in choosing the cheaper message, or $c(m) = c(m')$, and there exists a payoff-equivalent equilibrium in which we send the same message at $\mu^{-1}(m), \mu^{-1}(m')$. Therefore, suppose $\overline{a}(m) \neq \overline{a}(m')$. If $c(m) \leq c(m')$, the sender strictly prefers to send m on $(\overline{a}(m) - \varepsilon, \overline{a}(m) + \varepsilon) \in \mu^{-1}(m')$. If $c(m) > c(m')$ and $\overline{a}(m) < \overline{a}(m')$, the sender strictly prefers to send m' on $[s_3, s_4)$. If $c(m) > c(m')$ and $\overline{a}(m) > \overline{a}(m')$, the sender strictly prefers to send m' on $[s_1, s_2)$.

Case 2: Suppose $\overline{a}(m) \in [s_1, s_2)$. If there exists $m' \in \mu((s_2, s_3))$ with $c(m') \leq c(m)$, such that $\overline{a}(m') \in (s_2, s_3)$ then the sender strictly prefers to send m' on $[s_3, s_4)$. If $c(m') > c(m)$ for $m' \in \mu((s_2, s_3))$ then the sender strictly prefers to send m on $[s_2, s_3)$.

Case 3: $\overline{a}(m) \in [s_3, s_4)$. The proof is analogous to Case 2 and the lemma is proved.

■

Hence, the inverse of μ is a collection of disjoint intervals with the property that if $s, s' \in \mu^{-1}(m)$ for some m, so is $s'' \in (s, s,)$. Unless S indifferent between sending two signals at state s, then the same message is sent for some $(s - \varepsilon, s + \varepsilon)$ for some $\varepsilon > 0$.

Lemma 7 *If $b = 0$ then there are $n + 1$ solutions to* $\max_s(\overline{a}(m') - s')^2 + c(m')$.

Proof: Suppose that $U^S(\overline{a}, \widehat{m}, \underline{s}) > U^S(\overline{a}, \widehat{m}, \overline{s})$ where $\mu([\underline{s}, \overline{s})) = \widehat{m}$. As the distribution is uniform, $\overline{a} = \frac{s+\overline{s}}{2}$. This implies that $\left(\frac{s+\overline{s}}{2} - \underline{s}\right)^2 > \left(\frac{s+\overline{s}}{2} - \overline{s}\right)^2$, which cannot be the case. Combined with expression (A), we have $n + 1$ such solutions.

■

Proposition 1: If $b = 0$ then under Condition L an equilibrium always exists and it is exclusively the most informative, feasible equilibrium.

Proof: Suppose that expression (6) is satisfied for k. We need to check that it is not profitable for the $s = 0$ sender to transmit message a message of complexity $k + 1$. Because

k satisfied expression (6) it must be that

$$\begin{aligned}
\lambda_i^2 - \lambda_j^2 &= 4\left[c(z) - c(z-1)\right] \text{ for every } k \in \{1, ..., k\} \\
\text{where } j &\in \{\sum_{\beta=1}^{z-1} \gamma^{\beta-1}, ..., \sum_{\beta=1}^{z} \gamma^{\beta-1} - 1\} \text{ and } j \leq n = \sum_{\beta=1}^{k} \gamma^{\beta} \\
i &\in \{\sum_{\alpha=1}^{z-2} \gamma^{\alpha-1}, ..., \sum_{\alpha=1}^{z-1} \gamma^{\alpha-1} - 1\} \text{ and } i \geq 0
\end{aligned}$$

$$\lambda_0 + \lambda_1 + ... + \lambda_n = 1$$

$$\lambda_n > 0.$$

Therefore, $\lambda_0 = \sqrt{4c(k) + \lambda_n^2}$. And so the equilibrium payoffs for the S who received signal $s = 0$ is:

$$-\left(\frac{\lambda_0}{2} - 0\right)^2 - c(0) = -\left(\frac{\lambda_1}{2} - 0\right)^2 - c(1) = ... = -\left(\frac{\lambda_n}{2} - 0\right)^2 - c(k)$$

All of the messages used in equilibrium will not provide a profitable deviation, therefore we must use an out-of-equilibrium message to find a deviation. Any deviation accomplished by message of complexity $k+x$ where $x > 1$ can be accomplished with a lower communication cost by sending message of complexity $k+1$. Therefore, the cheapest (and therefore best candidate) out-of-equilibruim message is then the message with complexity $k+1$. If such a message is sent, R would have beliefs that the message was sent by state $s = 0$. Sending this signal yields a payoff of $-c(k+1)$. Therefore, the signal will be profitable when $\lambda_n^2 > 4\left[c(k+1) - c(k)\right]$.

For the case that $k+1$ is exactly feasible, it must be that $\lambda_{n+1} = 0$ and so $\lambda_n = 4\left[c(k+1) - c(k)\right]$. However, when $k+1$ is not feasible it must be that $\lambda_n^2 < 4c(k)$ and there is no profitable deviation. Therefore, when k is feasible and $k+1$ is not feasible, there is no profitable deviation to an equilibrium with a signal more complex than k and so a k-equilibrium always exists.

Now we will show that if k is feasible than there does not exist an equilibrium in which the most complex signal is less than k. Suppose that k is feasible and $k+1$ is not. Consider a candidate $k-1$-equilibrium. This candidate equilibrium is characterized by:

$$\widetilde{\lambda}_j^2 - \widetilde{\lambda}_i^2 = 4(c(i) - c(j) \text{ for } n' = \sum^{k-1} \gamma \geq i > j \geq 0$$

$$\text{where } i \in \{\sum_{\beta=1}^{k-1} \gamma^{\beta-1}, ..., \sum_{\beta=1}^{k} \gamma^{\beta-1} - 1\}$$

$$j \in \{\sum_{\alpha=1}^{k-2} \gamma^{\alpha-1}, ..., \sum_{\alpha=1}^{k-1} \gamma^{\alpha-1} - 1\}$$

$$\widetilde{\lambda}_{n'}^2 > 0$$

$$\widetilde{\lambda}_0 + \widetilde{\lambda}_1 + ... + \widetilde{\lambda}_{n'} = 1$$

When k is exactly feasible, a k-equilibrium would require:

$$\lambda_0 + \lambda_1 + ... + \lambda_n = 1.$$

where $\lambda_n = 0$ and $\lambda_{n-1} = 4\left[c(k) - c(k-1)\right]$. When k is strictly feasible, it must be that $\lambda_n > 0$ and

$$\lambda_{n-1} = 4\left[c(k) - c(k-1)\right] + \lambda_n^2 > 4\left[c(k) - c(k-1)\right].$$

Therefore, it must be that $\widetilde{\lambda}_{n'}^2 > 4\left[c(k) - c(k-1)\right]$ and that $\widetilde{\lambda}_0^2 > 4c(k)$. So we can write the equilibrium payoffs as:

$$U^S = -\left(\frac{\widetilde{\lambda}_0^2}{2} - 0\right)^2 < -c(k)$$

Deviation payoffs are $-c(k)$, therefore equilibrium payoffs are less than deviation payoffs and so a $k-1$-equilibrium cannot exist. Identical reasoning also rules out equilibria less complex than $k-1$.

To see that each k-equilibria uniquely determines the values of λ, we can rewrite

$$\lambda_0 + \lambda_1 + ... + \lambda_{n-1} + \lambda_n = 1$$

as

$$\begin{aligned} &2\sqrt{c(z) + \lambda_n^2} + 2\gamma\sqrt{c(z) - c(1) + \lambda_n^2} + 2\gamma^2\sqrt{c(z) - c(2) + \lambda_n^2} + ... \\ &+2\gamma^{z-1}\sqrt{c(z) - c(z-2) + \lambda_n^2} + 2\gamma^z\sqrt{c(z) - c(z-1) + \lambda_n^2} + \lambda_n = 1 \end{aligned} \quad (8)$$

The left hand side of expression (8) is strictly increasing in λ_n and therefore must only hold for a single value of λ_n. And so the proposition is proved.

■

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