CORRECTION TO "SEMI-CLASSICAL ANALYSIS OF SCHRÖDINGER OPERATORS AND COMPACTNESS IN THE $\overline{\partial}$-NEUMANN PROBLEM", JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS, 271 (2002), 267-282.

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We correct an inaccuracy in a proof in the above paper. Keep the notation from the paper (in particular regarding the L^2 and Sobolev norms). The inaccuracy occurs on page 278, where it was incorrectly stated that the term $\int_0^1 \|u\|_{-1,\,M_r}\,dr$ is estimated from above by $\|u\|^2_{-1,\,\Omega}$. This term should be estimated as follows. The surfaces M_r can be thought of as pieces of the boundaries of domains Ω_r which are sublevel sets of some smooth defining function for Ω. Denote by Δ^{-1} the inverse of the isomorphism $\Delta : W^1_0(\Omega) \to W^{-1}(\Omega)$. Then, $u - \Delta^{-1}(\Delta u)$ is harmonic, and Δ^{-1} is compact as an operator from $W^{-1}(\Omega) \to W^{3/4}(\Omega)$. For harmonic functions, the trace theorem (with loss of 1/2 derivative) holds for all Sobolev indices, whereas in general, it holds for indices greater than 1/2. Consequently, for all $\delta > 0$, there is a constant C_δ such that

$$\begin{aligned}\|u\|_{-1,\,M_r} \lesssim \|u\|_{-1,\,b\Omega_r} &\lesssim \|u - \Delta^{-1}(\Delta u)\|_{-1/2,\Omega_r} + \|\Delta^{-1}(\Delta u)\|_{3/4,\Omega_r}\\ &\lesssim \|u\|_{-1/2} + \|\Delta^{-1}(\Delta u)\|_{3/4}\\ &\lesssim \|u\|_{-1/2} + \delta\|\Delta u\|_{-1} + C_\delta\|\Delta u\|_{-3}.\end{aligned} \tag{1}$$

The last inequality results from a compactness estimate applied to $\Delta^{-1} : W^{-1}(\Omega) \to W^{3/4}(\Omega)$. Note that the constants in the first three inequalities may be taken to be independent of r.

Choosing $\delta = \varepsilon/C_\varepsilon$, we obtain as a replacement for the first estimate on page 278 that

$$\|u\|^2 \lesssim \varepsilon\left(\|L_1 u\|^2 + \|\bar{L}_1 u\|^2 + \|\Delta u\|^2_{-1}\right) + C_\varepsilon\|u\|^2_{-1/2}. \tag{2}$$

(Note that $\|\Delta u\|_{-3} \lesssim \|u\|_{-1} \lesssim \|u\|_{-1/2}$.) If $\alpha \in C^\infty_{(0,1)} \cap \mathrm{dom}(\overline{\partial}^*)$, then $\|\Delta\alpha\|^2_{-1}$ is dominated from above by $\|\overline{\partial}\alpha\|^2 + \|\overline{\partial}^*\alpha\|^2$. The argument now proceeds as on page 278: invoking maximal estimates to also dominate $\|L_1 u\|^2 + \|\bar{L}_1 u\|^2$ by $\|\overline{\partial}\alpha\|^2 + \|\overline{\partial}^*\alpha\|^2$, (2) gives

$$\|\alpha\|^2 \lesssim \varepsilon\left(\|\overline{\partial}\alpha\|^2 + \|\overline{\partial}^*\alpha\|^2\right) + C_\varepsilon\|\alpha\|^2_{-1/2}. \tag{3}$$

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Estimate (3) is essentially the required compactness estimate at the end of of the proof on page 278, except that $\|\alpha\|^2_{-1}$ there is now replaced by $\|\alpha\|^2_{-1/2}$. This change is inconsequential for the conclusion that the $\overline{\partial}$-Neumann operator is compact: any norm with respect to which the identity on L^2 is compact will do.

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