Written Assignment #1

Each question is worth 10 points. Please show all your work; answers without justification will not receive any credit. Upload a legible copy of your answers on Sakai by 11:59PM on November 2, 2017. If you are scanning a hand-written solution, please make sure that the scanned version is legible. Use a dark colored pencil or pen to write your answers; light pencil marks do not scan well.

1. [10 points] Suppose you want to design a hand-held video panel that is 5 inches by 6 inches and can display 100 pixels per inch along either side. If this panel is to support 24 bit color what is the smallest possible size (in bytes) needed for the panel’s frame buffer? Give your answer in general terms as well, i.e., if the panel is \( h \) inches by \( w \) inches and can display \( p \) pixels per inch along either side, express the smallest possible size of the frame buffer in terms of \( h \), \( w \) and \( p \).

2. [10 points] Given two vectors \( \vec{u} \) and \( \vec{v} \), show that the orthogonal projection (\( \vec{u}' \)) of \( \vec{u} \) onto \( \vec{v} \) (see figure below) is given by

\[
\vec{u}' = \frac{(\vec{u} \cdot \vec{v})}{(\vec{v} \cdot \vec{v})} \vec{v}
\]

3. [10 points] Given a unit cube with one corner at \((0,0,0)\) and the opposite corner at \((1,1,1)\), derive the transformations necessary to rotate the cube by \( \theta \) degrees about the main diagonal (from \((0,0,0)\) to \((1,1,1)\)) in the counterclockwise direction when we are looking along the diagonal toward the origin. You may express your answer as a concatenation of transformations (as we did in class). However, you must show how you arrived at your answer. A one-line statement expressing the answer will not receive any credit, even if it is correct.

4. [10 points] A very common low-level operation in computer graphics is testing if a point lies on or inside a triangle. One way to do this is by using what we know about affine combinations of points, described below (also discussed in class):

Given three points \( P_0, P_1, \) and \( P_2 \) (assume they are non-collinear, i.e., they do not all lie on a straight line), an affine combination of these points is the point \( Q \) given by \( Q = \alpha_0 P_0 + \alpha_1 P_1 + \alpha_2 P_2 \), where \( \alpha_0 + \alpha_1 + \alpha_2 = 1 \). \( Q \) lies on or inside the triangle defined by \( P_0, P_1 \) and \( P_2 \) if and only if each of \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) is \( \geq 0 \). (If any of them is < 0, then \( Q \) lies outside the triangle.)
Suppose $P_0 = (2, 1)$, $P_1 = (-1, 2)$ and $P_2 = (3, -1)$. How would you determine if an arbitrary point $S = (x, y)$ lies on or inside the triangle $\triangle(P_0, P_1, P_2)$? Derive the necessary inequalities that $x$ and $y$ must satisfy so that $S$ lies on or inside the triangle.

5. [10 points] In class, we derived the transformation matrices for translation, scaling and rotation. In addition, it was mentioned that shearing and reflection are also affine transformations. In this exercise, you are asked to derive a transformation matrix for reflecting a point about a line.

Consider the line going through the point $(0, c)$ (i.e., intersecting the $y$ axis at $y = c$) and making an angle $\theta$ (measured counter-clockwise) with the $x$ axis. Find the transformation matrix to reflect a point about this line. Express the answer as a $3 \times 3$ matrix, so that it can be applied to a column matrix in homogenous coordinates. Each matrix entry should be a function of $c$ and/or $\theta$. (Hint: What are the coordinates of a point $(x, y)$ when reflected about the $x$ axis?)