

CS 213

Homework Assignment 6

Given: February 26, 2009

Due: March 05, 2009

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

1. Show that Markov's inequality only applies to non-negative random variables. In other words, give an example of a random variable for which Markov's inequality gives a wrong answer.

Suppose Z is a random variable that is always at least -10 and has expectation 0. Since Z is negative Markov's inequality does not apply directly. Still, use Markov's inequality to show that the probability that $Z \geq 5$ is at most $2/3$.

2. Suppose we know that the average Business school graduate's salary is \$100,000.

- a. Knowing only this average, use Markov's inequality to find the best possible upper-bound for the fraction of Business school graduating students getting at least \$135,000.
- b. Demonstrate that this is the best possible bound by giving a distribution for which this bound holds with equality.
- c. Suppose you are now told that no student graduates with a salary less than \$70,000. how does this allow you to improve your previous bound? As before show that this is the best possible bound.

3. There are n locks and n matching keys for the locks, all mixed up. Each key fits exactly one lock. Our goal is to pair up the locks with their matching keys. In each operation you are allowed to take one key and one lock and to check if they fit. You can conclude that either you have found the matching lock for the key, or the lock is too big or too small. You are not allowed to compare the keys to keys or locks to locks. Design a randomized algorithm for solving this problem whose running time in expectation is $O(n \log n)$.

(*Hint:* Think quicksort.)