

# CS 213

## Homework Assignment 4

**Given:** February 12, 2009

**Due:** February 19, 2009

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

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- 1.** Given a directed graph  $G$  with  $n$  vertices represented using an  $n \times n$  adjacency matrix, give an algorithm that determines whether there is a node in  $G$  whose indegree is  $n - 1$  and outdegree is 0.
- 2.** Let  $G = (V, E)$  be a weighted, directed graph with weight function  $w : E \rightarrow \{0, 1, \dots, W\}$  for some non-negative integer  $W$ . Give an algorithm to compute the shortest paths from a given source vertex  $s$  in  $O(WV + E)$  time.
- 3.** Prove or disprove: If a directed graph  $G$  contains cycles, then the topological sort algorithm done in class produces a vertex ordering that minimizes the number of "bad" edges that are inconsistent with the ordering produced.
- 4.** Suppose that we are given a weighted, directed graph  $G = (V, E)$  in which edges that leave the source vertex  $s$  may have negative weights, all other edges weights are non-negative, and there are no negative weight cycles. Argue that Dijkstra's algorithm correctly finds shortest paths from  $s$  in this graph.