

CS 213

Homework Assignment 1

Given: January 22, 2009

Due: January 29, 2009

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit all your answers should be carefully justified. Each solution must be the student's own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

1. For each pair of expressions (A, B) in the table below, indicate whether A is $O, o, \Omega, \omega,$ or Θ of B . Assume that $k \geq 1, \epsilon > 0,$ and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

| A | B | O | o | Ω | ω | Θ |
|-------------|--------------|-----|-----|----------|----------|----------|
| $\lg^k n$ | n^ϵ | | | | | |
| n^k | c^n | | | | | |
| 2^n | $2^{n/2}$ | | | | | |
| $n^{\lg c}$ | $c^{\lg n}$ | | | | | |
| $\lg(n!)$ | $\lg(n^n)$ | | | | | |

2. Rank the following functions by order of their growth; i.e., find an arrangement g_1, g_2, \dots of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \dots$

| | | | |
|----------------------|----------------------|-----------------|---------------|
| $(\sqrt{2})^{\lg n}$ | n^2 | $n!$ | $(\lg n)!$ |
| $(3/2)^n$ | n^3 | $\lg^2 n$ | $\lg(n!)$ |
| 2^{2^n} | $n^{1/\lg n}$ | $\ln \ln n$ | $n \cdot 2^n$ |
| $n^{\lg \lg n}$ | $\ln n$ | 1 | $2^{\lg n}$ |
| $(\lg n)^{\lg n}$ | e^n | $4^{\lg n}$ | $(n+1)!$ |
| $\sqrt{\lg n}$ | $2^{\sqrt{2 \lg n}}$ | n | 2^n |
| $n \lg n$ | $2^{2^{n+1}}$ | $2^{100^{100}}$ | |

3. Assume that you have functions f and g such that $f(n) = O(g(n))$. For each of the following statements, decide whether you think it is true or false and give a brief supporting argument or a counterexample.

- a. $\lg f(n) = O(\lg g(n))$.
- b. $2^{f(n)} = O(2^{g(n)})$.
- c. $f(n)^2 = O(g(n)^2)$.