Problem Session 4

1. In how many ways can 3 boys and 3 girls sit in a row if
   
   (a) only the boys are allowed to sit together (i.e. no two girls are allowed to sit together?)
   (b) if no two people of the same sex are allowed to sit together?

2. There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?

3. Show that for $0 \leq k \leq n$
   \[ \sum_{m=k}^{n} \binom{m}{k} = \binom{n+1}{k+1} \]  
   \[ (1) \]

   Hint: Answer the question “How many subsets of size $k+1$ are contained in the set \{1, 2, \ldots, n+1\}?" in two different ways.