Problem Session 14

1. Prove that 6 \mid n(n^2 + 5), for all n \geq 1.

2. Problem 50.5 from the text, p. 427

Let G be an Eulerian graph. Prove that it is possible to partition the edge set of G such that the edges in each part of the partition form a cycle of G.

3. Problem 49.17 from the text, p. 421

Consider the following algorithm.
- Input: A connected graph G, with n vertices and m edges.
- Output: A spanning tree of G.
  1. Let T be a graph with the same vertices as G, but with no edges.
  2. Let e_1, e_2, ..., e_m be the edges of G.
  3. For k = 1, 2, ..., m, do:
     - If adding edge e_k to T does not form a cycle with edges already in T, then add edge e_k to T.
  4. Output T.

Prove that this algorithm is correct. In other words, prove that whenever the input to this algorithm is a connected graph, the output of this algorithm is a spanning tree of G.