1. Prove or disprove the following.
   (a) For all integers $n$, $n^3 - n$ is divisible by 3.
   (b) For all real numbers $x$, $2x^2 - 4x + 3 > 0$.

2. Prove that the following propositions are true.
   (a) The sum of any rational number and any irrational number is irrational.
   (b) $\sqrt{3}$ is irrational.

3. Suppose $a, b, x,$ and $y$ are integers. Prove that if $d|a$ and $d|b$, then $d|(ax + by)$.

4. Let $t$ be a positive integer. Prove the following statement by proving its contrapositive.
   
   if $r$ is irrational, then $r^{1/t}$ is irrational.

   Be sure to state the contrapositive explicitly.

5. Let $x_1, x_2, \ldots, x_n$ be $n$ real numbers. Let $\overline{x} = (x_1 + x_2 + \ldots + x_n)/n$ be their average. Use a proof by contradiction to prove that at least one of $x_1, x_2, \ldots, x_n$ is greater than or equal to $\overline{x}$.

6. Let $x, y,$ and $z$ be positive integers. Suppose that $x^2 + y^2 = z^2$. Prove the following.
   (a) if $z$ is even then both $x$ and $y$ are even.
   (b) $x$ and $y$ both cannot be odd.
   (c) if at least one of $x, y,$ and $z$ is odd then $z$ is odd.
7. Suppose we have three people, Alice, Bob and you. Alice and Bob have both taken CS 171 and got an “A+”, so their logical reasoning is both flawless and instantaneous. There are three red hats and two white hats. Each of the three people wear one hat. Each person can neither see their own hat nor the unused hats. However, each of the three persons can see the hats worn by other two people. When Alice is asked “What is the color of your hat?”, she replies, “I don’t know.”. Next, Bob is asked “Now, Bob, what is the color of your hat?”, he replies, “I don’t know.”. At this point can you tell the color of your hat? If so, what color is it? Explain your answer.