Given: February 28, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student’s own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

1. Here are seven propositions:

\[
\begin{align*}
\neg x_1 \lor x_2 \lor x_7 \\
\neg x_5 \lor x_6 \lor x_7 \\
x_2 \lor \neg x_4 \lor x_6 \\
x_4 \lor x_5 \lor x_7 \\
x_3 \lor \neg x_5 \lor x_8 \\
x_9 \lor \neg x_8 \lor x_2 \\
\neg x_3 \lor x_9 \lor x_4 
\end{align*}
\]

Note that

1. Each proposition is the OR of the three terms of the form \(x_i\) or of the form \(\neg x_i\).

2. The variables in the three terms in each proposition are all different.

Suppose that we assign true/false values to the variables \(x_1, \ldots, x_9\) independently and with equal probability.

a. What is the probability that a single proposition is true?

b. What is the expected number of true propositions?

c. Use your answer to prove that there exists an assignment to the variables that makes all of the propositions true.

2. A monkey types on a 26-letter keyboard that has lowercase letters only. Each letter is chosen independently and uniformly at random from the alphabet. If the monkey types 1,000,000 letters, what is the expected number of times the sequence “math” appears?

3. Eight men and seven women are randomly assigned distinct numbers from 1 to 15. On average, how many pairs of consecutive numbers are assigned to people of opposite sex?