Given: January 26, 2012  Due: January 31, 2012

This assignment is due by the end of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be the student’s own work. Assistance should be sought or accepted only from the course staff. Any violation of this rule will be dealt with harshly.

1. Answer each of the following questions. You don’t need to show your work.

Questions a-e are based on the scenario that a president, treasurer, and secretary, all different, are to be chosen from a club consisting of 10 people. Calculate the number of different choices of officers possible if

a. there are no restrictions.

b. A and B will not serve together.

c. C and D will serve together or not at all.

d. E must be an officer.

e. F will serve only if he is president.

f. Five separate awards are to be presented to selected students from a class of 30. How many different outcomes are possible if a student can receive any number of awards?

g. In the above scenario, how many outcomes are possible if each student can receive at most one award?

2. There are 15 different air conditioning vents in a movie theater. To keep the air fresh, at least one of the vents has to be on at all times. In how many ways can this be done?

3. A tennis tournament has $2n$ contestants. We want to pair them up for the first round of singles matches. Show that the number of different ways in which first round matches can be conducted is $\frac{(2n)!}{2^n \times n!}$.

4. A committee of five is to be chosen to represent the psychology department that has 22 faculty - 10 men and 12 women. How many ways can the committee be formed if it is to contain at least two women?
5. A coin is flipped 15 times where each flip comes up either heads or tails. How many possible outcomes (a) contain exactly four tails?, (b) contain at least three heads?

6. How many strings are there of four lower-case letters that have the letter $x$ in them?