1. Let $G = (V, E)$ be a connected graph that is not Eulerian. Prove that it is possible to add a single vertex to $G$, together with some edges from this new vertex to some old vertices such that the new graph is Eulerian.

2. An Eulerian walk in a graph is a walk that traverses each edge exactly once and returns to a different vertex than where it started from. Prove that a connected graph has an Eulerian walk if and only if it has exactly two vertices of odd degree.

3. The complement of a graph $G$ is a new graph formed by removing all the edges of $G$ and replacing them by all possible edges that are not in $G$. Formally, consider a graph $G = (V, E)$. Then, the complement of the graph $G$ is the graph $\overline{G} = (V, \overline{E})$, where

$$\overline{E} = \{\{x, y\} | x \neq y, \{x, y\} \notin E\}$$

Prove that for any graph $G$, $G$ or $\overline{G}$ (or both) must be connected.