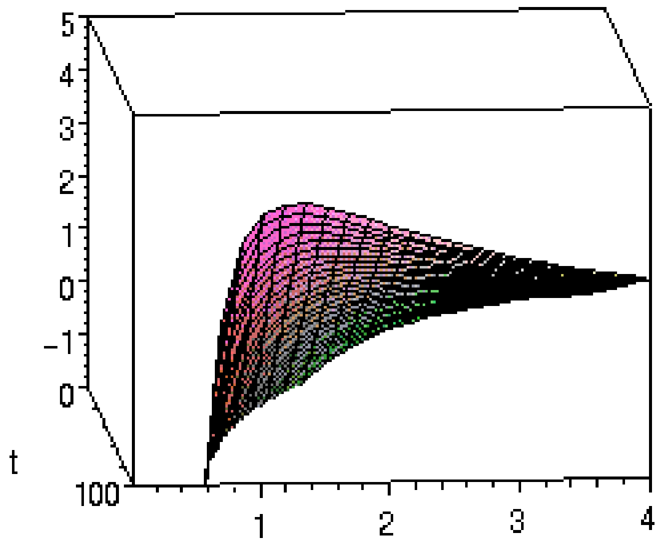
Taylor Series Lab



Websites www.maplesoft.com, www.maple4students.com

start xmaple

xmaple &

Inside the main Maple window there are two smaller windows; a (green) Help window and a blank (white) worksheet where you can enter Maple commands. Minimize the Help window for now. Note the help-menu on the right. You can get *context-sensitive help* by highlighting a single word and clicking on help.

Examples of the maple syntax

$f := x \to x;$

$g := x \to x - x^3/3!;$

smartplot(sin(x),f(x),g(x));

Right-click on the plot to alter the range of the X and Y axes.

plot({sin(x),f(x),g(x)},x=-2*Pi..2*Pi,axes=boxed,title="test plot");

Legends Under the **options** pull-down menu at the top of the page, select "Display 2-D Legends".

Save your work (.mws format) often; xmaple tends to crash!

Convergence of 1-D Taylor Series

In this section you explore the converge of Taylor series for smooth functions like $\sin(x)$, and for functions with singularities such as $\ln(1+x)$.

1. Using pen and paper, evaluate the Taylor series for sin(x) up to x^9. Include the answer in your workbook. Note that the Maple syntax for 10 factorial is just 10!.

2. Plot the linear, quadratic, cubic,... Taylor series up to the 9^{th} power of x on the same graph using **smartplot**. Also, plot $sin(x)$ on the same graph.

3. Do the polynomials converge to $\sin(x)$?

4. Repeat steps 1$\rightarrow$ 2 above for $\ln(1+x)$.

5. Do the polynomials converge to $\ln(1+x)$?

6. In order for the Taylor series to give an accurate representation of the function, it is essential that the terms which are neglected (beyond x^9) are small. Examine the explicit expressions that you derived for the Taylor series of $\sin(x)$ and $\ln(1+x)$. How rapidly do the Taylor series coefficients decrease in size as you move from the first coefficient to the coefficient of x^9? Do the Taylor series coefficients for $\sin(x)$ and $\ln(1+x)$ decrease at the same rate, or do the coefficients for one series decay more rapidly? Based on the rate of decay of the terms, which Taylor series do you expect to be more accurate, the series for $\sin(x)$ or for $\ln(1+x)$?

7. Repeat the plots for the sin function using the advanced/faster **mtaylor** syntax. For example:

   $$f := x- > mtaylor(sin(x), x = 0.0, 4);$$

   smartplot(sin(x),f(x));

   This gives a Taylor series up to the fourth power of x, expanded about the origin $x = 0.0$. You can edit the Legend by right-clicking on the graph.

8. Change the origin of the taylor series from 0 to Pi/2 using:

$$f := x- > mtaylor(sin(x), x = Pi/2, 4);$$

 and redo the plot for the Taylor series up to the 4^{th} power in x. How does the 4^{th} order polynomial change when the origin is shifted from 0 to Pi/2?

9. Use the mtaylor function to expand $\sin(x)$ in a taylor series up to the 20^{th} term. Plot this taylor series, together with the exact function $\sin(x)$. Edit the Legend to indicate which is which. How do they compare?

10. Repeat this, but with the origin of the Taylor series shifted from 0 to Pi/2. How much difference does shifting the origin make?

11. Expand $\ln(1 + x)$ in a Taylor series up to the 20^{th} term, and plot this Taylor series together with the exact function $\ln(1+x)$. Edit the Legend to indicate which is which. How do they compare?

12. Repeat this, but with the origin of the Taylor series shifted from 0 to Pi/2. How much difference does shifting the origin make?

13. Expand $\tan(x)$ in a Taylor series up to the 20^{th} term, and plot this Taylor series together with the exact function $\tan(x)$. How good is the polynomial approximation in regions where $\tan(x)$ is large or infinite. What about in other areas?

Application of Multi-Variable Taylor Series to Thermodynamics

1. Construct the ideal-gas function

   $$P(V,T) = RT/V$$

   using the maple syntax,

   $$p := (v,t)- > 8.31 * t/v;$$

2. Plot the function using:

   smartplot3d(p(v,t))

   Add axes to the plot by right-clicking, and adjust the range of the plot to physically-reasonable values. Temperature and Pressure should be positive, and also use a minimum volume of 0.01. You can rotate the plot using the left mouse button.

3. Now expand $p(v,t)$ in a 2-D Taylor series in v and t using mtaylor. You can't expand this function about $v = 0$ (**why not?**), so expand about the point $(v = 5, t = 0)$,

   $$f := (v,t)- > mtaylor(p(v,t), [v = 5, t = 0], 3);$$

   See help mtaylor for details.

4. Compare the exact function with the Taylor series using either smartplot3d or plot3d,

   $$e := (v,t)- > f(v,t) - p(v,t);$$

   smartplot3d(e(v,t));

   OR plot3d(e(v,t),t=0..1000,v=0.1..10,view=0..10000,axes=boxed);

5. Investigate how well the Taylor series converges to the exact function as you increase the number of terms in the series, and as you change the origin of the series. Where is the greatest error? Is this what you expected?

6. Now redo the above for a non-ideal gas ('Van der Waals' gas), which accounts for the volume of the atoms and (more importantly) the attractive forces between atoms,

$$p := (v,t)- > 0.083 * t/(v - 0.03) - 5.5/v^2;$$

$$f := (v,t)- > mtaylor(p(v,t),[v = 5, t = 0], 3);$$

$$e := (v,t)- > f(v,t) - p(v,t);$$

smartplot3d(p(v,t))
smartplot3d(e(v,t));
OR
plot3d(e(v,t),t=570..650,v=0.03..0.1,view=0..500,axes=boxed);

where 0.03 is the volume of the atoms and 5.5 is the Van der Waals force of attraction between the atoms.

7. Can you find the point(s) at which the Van der Waals gas condenses from a gas to a liquid? (**hint:** Set the range of V to be 0.03 to 4. Set the range of T to be 0 to 100, and the range of P to be -2 to 2. Then align the graph so that the V axis is going from left to right. Is P always a decreasing function of V???) Does the Taylor series work well in this region? (**hint:** use the same ranges of T and V as above).

Lab Report

Save you maple output as a .mws file and email it to me. Also, include in the email answers to the questions above.

Bonus Project

Get maple to solve for volume v as a function of t and p.

Other uses for maple

In addition to plotting functions, maple is also very useful for doing calculus (derivatives and integrals).