Approximating low-congestion routing and column-restricted packing problems

Alok Baveja* and Aravind Srinivasan

School of Business, Rutgers University, Camden, NJ 08102, USA
Bell Laboratories, Lucent Technologies, 600-700 Mountain Ave., Murray Hill, NJ 07974-0636, USA

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Abstract

We contribute to a body of research asserting that the fractional and integral optima of column-sparse integer programs are “nearby”. This yields improved approximation algorithms for some generalizations of the knapsack problem, with applications to low-congestion routing in networks, file replication in distributed databases, and other packing problems.

Keywords: Algorithms; Approximation algorithms; Packing; Integer programming; Routing

1. Introduction

Let \( Z^+ \) denote the set of non-negative integers, \( v^T \) the transpose of a (column) vector \( v \), and \([k] \doteq \{1, 2, \ldots, k\} \). A key family of packing problems that includes classical NP-hard problems such as knapsack, independent sets in graphs, matchings in hypergraphs etc., has been introduced in [5]. These problems, named column-restricted packing integer programs (CPIPs) in [5], are integer programs of the form “maximize \( w^T x \), subject to \( Ax \leq b \) and \( x_j \in \{0, 1, \ldots, d_j\} \) for each \( j \)” (\( \forall j, d_j \in Z^+ \)); all entries of the matrix \( A \) and of the vectors \( b \) and \( w \) are non-negative. Also, all nonzero entries in any given column of \( A \) are the same, hence “column-restricted”. Suppose, e.g., that we have files \( F_1, \ldots, F_n \) with \( F_j \) having size \( \rho_j \), and \( m \) servers, each having some capacity. If \( F_j \) is selected, it is to be placed on a specified subset \( S_j \) of the \( m \) servers; the total load on any given server should not exceed its capacity. Given a benefit for each \( F_j \), the problem of selecting a subset of the files that maximizes the total benefit subject to the above constraints, is a CPIP with \( d_j = 1 \) for all \( j \). Since CPIPs are NP-hard, there is much interest in developing approximation algorithms for them. The best general provable approximations to-date for such packing problems start by considering the linear programming (LP) relaxation where each \( x_j \) is relaxed to be a real lying in \([0, d_j] \); the objective function value of an optimal solution of this LP upper-bounds the optimal objective function value of the CPIP. The crucial step then is to show how to “round” the LP solution...